

Application of advanced surrogate models in structural uncertainty quantification and reliability analysis

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Abstract. Modern engineering structures are developing toward larger scales, greater complexity, and harsher environments, posing significant challenges to structural reliability analysis. Traditional methods often suffer from high computational costs and limited accuracy when dealing with highly nonlinear problems, multidimensional uncertainties, and expensive simulations. To address these challenges, this study develops an efficient and accurate framework for structural reliability analysis by integrating optimization algorithms, active learning, multifidelity information fusion, ensemble modeling, and physics-informed learning. The main contributions are as follows: (1) A first-order reliability method based on an augmented Lagrangian penalty function and an enhanced sparrow search algorithm is proposed to improve convergence and avoid local optima and numerical ill-conditioning in complex nonlinear reliability problems. (2) A hybrid active-learning strategy is developed for Kriging modeling, balancing global exploration and local exploitation while directing samples toward high-risk and informative regions. (3) A multifidelity active Kriging method is established by effectively integrating computationally efficient low-fidelity models with accurate high-fidelity simulations, significantly reducing computational costs. (4) An entropy-weighted ensemble surrogate model is developed to improve prediction stability and accuracy by adaptively combining multiple surrogate models according to their performance and uncertainty. (5) A physics-informed neural network-based reliability and optimization framework is proposed, embedding governing equations and boundary conditions into the loss function to enforce physical constraints and ensure physically feasible optimal solutions. The proposed methods provide effective tools for reliability-based design of complex engineering systems, including deep-water risers.

1. Introduction

The cost of structural reliability analysis is closely tied to the number of times the response model must be evaluated. Material properties, dimensions, loads and boundary conditions are uncertain, and a single deterministic calculation cannot describe their effects on the probability of failure. For nonlinear structures with many uncertain inputs, reliability estimation may require a substantial number of response calculations. Detailed finite element models make these calculations expensive, especially in uncertainty-based design optimization, where structural responses and reliability constraints must be reassessed as the design changes [1].

Replacing some of the model evaluations with surrogate predictions can make the calculation more manageable. Kriging, support vector regression, polynomial chaos expansions, response surface

methods and neural networks have been used for this purpose [2-10]. Kriging-based surrogate strategies have also been incorporated into reliability-based design optimization of offshore wind turbine support structures, towers and monopiles, where repeated high-cost structural analyses make adaptive model construction particularly useful [11-13]. Support vector regression has likewise been used to construct efficient response approximations for fatigue reliability assessment of offshore wind turbine support structures under hybrid uncertainty [14]. In a reliability calculation, the location of a prediction error deserves particular attention. A surrogate may reproduce its training data well but still assign the wrong failure status to samples near the limit-state boundary, thereby affecting the estimated failure probability. Adding samples in this region helps refine the boundary, although the search must also cover parts of the input space where failure regions have not yet been identified. Active learning uses information from the current surrogate to guide this process, balancing the search for relevant regions with further sampling around the estimated boundary [15-18].

The expense of generating training responses remains a concern even when samples are selected adaptively. Low-fidelity models provide cheaper responses, though these may differ systematically from high-fidelity results. By using information from both, multi-fidelity modelling reduces the number of high-fidelity evaluations needed to build the surrogate [19-24]. Choosing the surrogate itself is less straightforward: performance varies between problems and can also vary within the input space of a single problem. This has motivated ensemble methods, in which predictions from several models are combined to reduce reliance on any one approximation [25-28]. For structures described by known governing equations and boundary conditions, physical information can also contribute to model training. Physics-informed neural networks incorporate these relations into the training loss, providing constraints on the predicted response in addition to those supplied by the data. More broadly, recent studies of AI-assisted fatigue analysis also indicate a shift from purely data-driven models toward approaches that combine physical knowledge with learned representations [29].

This study addresses these computational and modelling difficulties through five related developments:

1. An augmented Lagrangian penalty formulation is combined with enhanced Sparrow Search Optimization to locate the most probable point in nonlinear reliability problems.
2. A hybrid active-learning strategy alternates between global sampling and local refinement, improving the Kriging approximation in regions relevant to the reliability calculation.
3. An adaptive multi-fidelity Kriging method uses low-fidelity information together with selected high-fidelity evaluations to reduce the cost of surrogate construction.
4. An entropy-weighted ensemble combines surrogate predictions using information about model behaviour across the sample set and local prediction uncertainty.
5. A physics-informed neural-network framework incorporates governing equations and boundary conditions into response modelling and is integrated with reliability analysis and reliability-based design optimization.

The methods are presented in the following sections. A deep-water riser design problem is then used to illustrate the physics-informed optimization framework and compare its computational cost and design results with those of the alternative solution methods.

2. Augmented Lagrangian-Based FORM with ESSO

In the first-order reliability method (FORM), the most probable point (MPP) lies on the limit-state surface. Searching for this point is a constrained optimization problem. The non-gradient optimizer used here requires an unconstrained objective, so the limit-state constraint is incorporated into a penalty formulation. A conventional exterior penalty introduces a factor whose value is usually chosen from experience. An unsuitable choice can affect the accuracy of the solution or cause numerical ill-conditioning. The present search instead uses the augmented Lagrangian penalty function (ALPF) in Eq. (1) as its fitness function [30].

$$L(x, u, v, \rho) = \|u\| + v(g(x)) + \frac{1}{2\rho} (g(x))^2 \quad (1)$$

The Lagrange multiplier and penalty parameter are updated as the calculation proceeds. This reduces the dependence on a penalty factor selected manually before the search.

$$\begin{aligned} v_{k+1} &= v_k + \frac{g(x_{k+1})}{\rho} \\ \rho_{k+1} &= \rho_K \sqrt{\max(x_{k+1}) + 1} \end{aligned} \quad (2)$$

The formulation has been examined for different classes of optimization problem. For a non-convex problem, a duality gap can prevent the optimum from being recovered directly. The penalty contribution is adjusted over successive iterations through the parameter update in Eq. (2), rather than being fixed throughout the search.

Enhanced Sparrow Search Optimization (ESSO) is used to solve the unconstrained problem. The original Sparrow Search Optimization (SSO) can have difficulty finding the relevant region of a complex search space and then refining the solution within it. ESSO adds position adjustments and a search around the best solution found so far. These adjustments use normally distributed random perturbations. At an update, the algorithm probabilistically retains the position proposed by SSO, adjusts that position, or perturbs the global-best position.

$$x_{i,j}^{k+1} = \begin{cases} x_{i,j}^{k+1} \left[1 + 0.1\gamma^2 \times \text{norm}(0,1) \right] & r < p \\ x_{i,j}^{k+1} & r_1 \geq p \end{cases} \quad (3)$$

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$$|\beta_k - \beta_{k-1}| < \varepsilon_{t3} \quad (4)$$

If this change exceeds the prescribed tolerance, the ALPF parameters are updated and ESSO is run again. The calculation stops when the criterion is met, and returns the final reliability index together with the corresponding MPP.

Tests reported for the method cover eight numerical problems and three engineering applications. In these comparisons, the ALPF formulation improves the accuracy of the calculation, and ESSO makes the search for the reliability index more robust, with better global convergence.

3. Hybrid Active-Learning Strategy for Kriging-Based FORM

In FORM, the accuracy of the reliability result depends on locating the most probable point (MPP) accurately [31]. This does not necessarily require a highly accurate Kriging model throughout the entire design space: regions far from the MPP may have little influence on the search, and refining the model everywhere can increase the number of limit-state evaluations. A purely local sampling strategy, however, may miss the relevant failure boundary if the current MPP is a local optimum. The original study therefore introduces a hybrid strategy that uses both global and local sampling to construct the active Kriging model.

The strategy uses a trade-off factor to decide whether a Kriging update should draw candidate points from the global design space or from a local region around the current MPP. The factor depends on the optimization iteration k and the number of Kriging updates aa made during that iteration.

$$\gamma = \text{mod}(aa, k+1) \quad (5)$$

When the factor selects global sampling, a candidate set is generated over the design space using Latin hypercube sampling (LHS). When it selects local sampling, candidates are generated around the current

MPP. Following the local Kriging modelling approach, the local search region is centred at the current MPP, with its radius typically ranging from 1.2 to 1.5 times the current reliability index [32]. A radius of 1.5 times the current reliability index is adopted in this study. As the optimization proceeds, global updates are more likely in earlier iterations, while local updates become more likely later. The maximum number of Kriging updates in one optimization iteration is set to $aa_{\max}=10$ in the original study to prevent the update process from becoming trapped in repeated Kriging cycles.

The calculation starts with a Latin hypercube design of experiments (DOE). Each point is evaluated with the original limit-state function, and the resulting responses are used to fit the first Kriging model. ESSO then searches this approximation for the current MPP. If the model does not meet the prescribed accuracy criterion, the trade-off factor selects either the global candidate region or the local region around that point. A learning function chooses one candidate for evaluation with the original limit-state function. Its response is added to the DOE, the Kriging model is fitted again, and the MPP search continues. Both the model-accuracy condition and the reliability-index stopping condition must be satisfied before the result is accepted.

In the numerical and engineering examples reported for this strategy, the combined global and local sampling procedure resolves the limit-state surface with fewer calls to the finite element model. The reduction in those calls accounts for the saving in model-construction cost.

4. Multi-Fidelity Active Kriging-Assisted FORM

High-fidelity simulations can provide accurate structural responses but may require substantial computational resources. Low-fidelity models are less expensive to evaluate, although their predictions may contain systematic bias. The original study addresses this cost–accuracy trade-off by developing a multi-fidelity (MF) Kriging model that uses many low-fidelity samples together with a smaller set of high-fidelity samples. The model is then used within FORM to estimate the reliability index and locate the MPP.

The method considers two levels of fidelity. The low-fidelity dataset is denoted by (X_l, Y_l) and the high-fidelity dataset by (X_h, Y_h) . The original study constructs the high-fidelity DOE by selecting a subset from the low-fidelity DOE using an exchange algorithm and the Morris–Mitchell criterion [33].

$$X = \begin{bmatrix} X_l \\ X_h \end{bmatrix} = \begin{bmatrix} x_l^1, \dots, x_l^{N_l}, x_h^1, \dots, x_h^{N_h} \end{bmatrix} \quad (6)$$

$$Y = \begin{bmatrix} Y_l \\ Y_h \end{bmatrix} = \begin{bmatrix} f_l(x_l^1), \dots, f_l(x_l^{N_l}), f_h(x_h^1), \dots, f_h(x_h^{N_h}) \end{bmatrix} \quad (7)$$

The MF-Kriging surrogate uses an additive discrepancy correction. A low-fidelity Kriging model is first fitted to the low-fidelity dataset. At each high-fidelity training location, its prediction is subtracted from the high-fidelity response to obtain a discrepancy target. A second Kriging model is fitted to these targets. Its prediction is added to the low-fidelity prediction in Eq. (8). The difference in Eq. (9) therefore defines the discrepancy training data at the high-fidelity sample locations, rather than the discrepancy predictor at an arbitrary input.

$$\hat{g}_m(x) = \hat{g}_l(x) + \hat{\delta}(x) \quad (8)$$

$$\hat{\delta}(x) = Y_h - \hat{g}_l(x) \quad (9)$$

The low-fidelity Kriging model is first trained using the low-fidelity dataset (X_l, Y_l) . At each high-fidelity sample location, the discrepancy target is obtained from the difference between the corresponding high-fidelity response and the prediction of the low-fidelity Kriging model. These discrepancy values are then used to train a second Kriging model, whose prediction is denoted by $\hat{\delta}(x)$. The final MF-Kriging prediction in Eq. (8) is obtained by adding this discrepancy prediction to the low-fidelity prediction. Therefore, the quantity defined in Eq. (9) represents the discrepancy training information at the high-fidelity sample locations, whereas $\hat{\delta}(x)$ in Eq. (8) denotes the prediction of the trained discrepancy model at an arbitrary input.

For adaptive updating, the study introduces three multi-fidelity learning functions: the multi-fidelity U function (MU), the multi-fidelity expected feasibility function (MEFF), and the multi-fidelity H function (MH). These extend the corresponding single-fidelity learning functions to account for the different prediction characteristics and evaluation costs of the two models.

$$MU(x, m) = \frac{U(x, m)v(x, m)}{s(m)}, m = l, h \quad (10)$$

$$v(x, m) = \sqrt{\frac{p_h(x)p_m(x)}{(k_h(x) - k_m(x)) + 1}}, m = l, h \quad (11)$$

$$s(m) = \frac{s(h)}{s(l)} \quad (12)$$

In Eqs. (10)-(12), x denotes a candidate sample and m denotes its fidelity level, with $m = l$ for the low-fidelity model and $m = h$ for the high-fidelity model. $U(x, m)$ is the conventional U learning-function value evaluated using the Kriging model corresponding to fidelity m . The factor $v(x, m)$ is introduced to characterize the predictive relationship between the selected fidelity model and the high-fidelity model. The quantities involved in $v(x, m)$ are obtained from the Kriging predictions and their associated predictive uncertainties at the candidate point. The resource factor $s(m)$ represents the relative computational cost of evaluating fidelity m . The high-fidelity evaluation is used as the reference cost, while the low-fidelity cost is expressed relative to it. Thus, when a low-fidelity evaluation requires 10% of the computational cost of a high-fidelity evaluation, the corresponding relative cost is 0.1.

$$MEFF(x, m) = \frac{EFF(x, m)v(x, m)}{s(m)}, m = l, h \quad (13)$$

$$MH(x, m) = \frac{H(x, m)v(x, m)}{s(m)}, m = l, h \quad (14)$$

MU, MEFF and MH are alternative acquisition rules. Each rule searches over candidate locations and the two fidelity levels. The selected pair minimises MU or maximises MEFF or MH, depending on the rule used. The response is evaluated at the selected fidelity and added to the corresponding dataset before the surrogate is updated. The rules are not combined into a single score.

$$(x_{new}, m_{new}) = \begin{cases} \arg \min (MU(x, m)) \\ \arg \max (MEFF(x, m)) \\ \arg \max (MH(x, m)) \end{cases} \quad (15)$$

During reliability analysis, the MF-Kriging prediction replaces the original limit-state function in the FORM optimization. The model is updated with newly selected samples, and its accuracy is checked using the stopping criterion in the original study. The reliability optimization then proceeds using the ALPF formulation described in the preceding section. If the optimization or model-accuracy condition is not satisfied, the corresponding parameters or training data are updated and the process continues until the stopping conditions are met. The MPP and reliability index are then reported.

5. Entropy-Weighted Ensemble Surrogate Modelling

The predictive performance of a surrogate model varies with the characteristics of the reliability problem. A single model may therefore have difficulty maintaining sufficient accuracy across different nonlinear response patterns. To address this limitation, an ensemble strategy is developed using the entropy weight method (EWM). Kriging and an artificial neural network (ANN) are used as the constituent models. Their contributions are determined by combining information from the overall sample set with local prediction-variance estimates.

The initial design of experiments (DOE) is generated using Latin hypercube sampling, and the corresponding responses are evaluated using the original limit-state function. Both constituent models are trained on the current DOE. The ensemble prediction is then obtained by combining their individual predictions with the assigned weights.

$$\hat{y}(x) = \sum_i^{N_M} w_i(x) \hat{y}_i(x) \quad (16)$$

Local weighting methods, including local best surrogate and local weighted average surrogate approaches, use information about local predictive performance [28]. The entropy calculation adds information about the distribution of each model's predictions across a common evaluation set. This information is distinct from prediction error and does not, on its own, establish which model is more accurate.

Both constituent models are evaluated at the same set of input points. Each row of the prediction matrix corresponds to an evaluation point, and each column corresponds to a model. Its entries are predicted limit-state values. Standardisation and the subsequent proportion calculation are performed separately for each column before the model entropy is evaluated.

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1N_M} \\ \vdots & \ddots & \vdots \\ a_{k1} & \cdots & a_{kN_M} \end{bmatrix} \quad (17)$$

$$B = \left[b_{ij} = \frac{a_{ij}}{\sqrt{\sum_{i=1}^n a_{ij}^2}} \right], (i=1,2,\dots,k; j=1,2,\dots,N_M) \quad (18)$$

$$p_{ij} = \frac{b_{ij}}{\sum_{i=1}^n b_{ij}}, (i=1,2,\dots,k; j=1,2,\dots,N_M) \quad (19)$$

In Eqs. (17)-(19), k denotes the number of samples used for the entropy calculation and N_M denotes the number of constituent surrogate models. The element a_{ij} is the prediction of the j -th surrogate model at the i -th sample point. The prediction matrix A is first standardized to obtain $B = [b_{ij}]$, and the normalized proportion p_{ij} is then calculated for each sample under each surrogate model. These proportions are subsequently used to evaluate the entropy of each constituent model.

$$\zeta_j = -\frac{1}{\ln(k)} \sum_{i=1}^k p_{ij} \ln(p_{ij}), (i=1,2,\dots,k; j=1,2,\dots,N_M) \quad (20)$$

$$G_j = 1 - E_j \quad (21)$$

$$\omega r_j = \frac{G_j}{\sum_{j=1}^l G_j} \quad (22)$$

Each model receives a final weight calculated from its entropy weight and local prediction variance. The prediction information needed for this calculation comes from cross-validation and jackknife calculations. If a model does not provide a prediction variance, a jackknife approximation is used. As new samples are added to the training set, the models are retrained and their weights are calculated again.

$$w_j(x) = \omega r_j \frac{\sum_{i=1, i \neq j}^{N_M} e_i(x)}{(N_M - 1) \sum_{i=1}^{N_M} e_i(x)} \quad (23)$$

The final model weight combines the global entropy weight with local predictive information. In Eq. (23), $e_j(x)$ denotes the local prediction-error measure associated with the j -th constituent surrogate at the candidate point x . The entropy component reflects the overall behaviour of the model across the sample set, whereas the local component adjusts the weight according to the model behaviour near the candidate point. When a constituent model does not directly provide the predictive uncertainty required by the learning procedure, a jackknife approximation is used.

For rare failure events, a conventional Monte Carlo sample set may contain too few points near the limit-state surface to give the learning function useful candidates. Successive conditional sample sets are therefore constructed following the idea of subset simulation (SS). Markov chain Monte Carlo sampling is used to generate these sets. In the reported comparison of sample distributions, the modified Metropolis–Hastings delayed rejection (MMHDR) algorithm gave better coverage of the relevant limit-state region than the other strategies tested. It was selected on this basis. The samples from the last two levels are combined into a local candidate set, from which the learning function chooses the next update point.

The ensemble is used within a Monte Carlo reliability-analysis framework. Its predictions over the Monte Carlo sample set provide the current failure-probability estimate. The selected update point is evaluated using the original limit-state function and added to the DOE. The constituent models are retrained, their weights are recalculated, and the ensemble prediction is updated. Model updating terminates when the original stopping criterion, which compares the failure-probability estimates of the constituent models with that of the ensemble, is satisfied.

$$\frac{e_{P_F}}{P_F} < \varepsilon_t \quad (24)$$

$$e_{P_F} = \max \left\{ |P_F - P_{F,i}|, |P_F - P_{F,i+1}|, \dots, |P_F - P_{F,n}| \right\}$$

In Eq. (24), $P_{F,i}$ denotes the failure probability estimated by the i -th constituent surrogate model, while P_F denotes the failure probability estimated by the ensemble surrogate. The maximum discrepancy between the ensemble estimates and the individual-model estimates is used to evaluate convergence. The surrogate-updating process is terminated when the relative discrepancy is smaller than the prescribed threshold ε_f , otherwise, a new sample is selected and the ensemble model is updated.

The original numerical comparisons show that the ensemble approach requires fewer limit-state evaluations than the individual surrogate models. The proposed EWM strategy also achieves improved accuracy with fewer evaluations than the investigated local weighting methods in those numerical tests. The engineering examples demonstrate its computational-efficiency benefits while retaining useful reliability-estimation accuracy. In the composite-laminate example, the local weighted average method produces a more accurate failure-probability estimate, whereas EWM requires fewer limit-state evaluations.

6. Physics-Informed Neural Networks for Reliability Analysis and Design Optimization

For structural systems governed by differential equations, physics-informed neural networks (PINNs) incorporate the governing physical relationships into model training. This reduces the dependence on response data required by conventional data-driven neural networks. Physics-informed learning has also been applied to fatigue crack-growth assessment, where mechanical knowledge can be incorporated into the network while reducing exclusive dependence on experimental data [34]. The proposed framework combines adaptive loss weighting, a modified fully connected architecture, and active learning to construct PINNs for reliability analysis. The trained models are also incorporated into a decoupled reliability-based design optimization (RBDO) procedure.

The network approximates the structural response as a function of the uncertain inputs and the relevant spatial and temporal coordinates. Its output can be modified to satisfy the prescribed initial and Dirichlet boundary conditions through the output transformation given in the original formulation.

$$\hat{u}(\xi, x, t) = u_b(x_b, t_i) + B \cdot u_{NN}(x, t, \xi) \quad (25)$$

The governing differential equation is evaluated using the network prediction and its derivatives, which are obtained through automatic differentiation. The resulting equation residuals are used to construct the training loss. Network parameters are then determined by minimizing this loss.

$$Loss_p = \frac{1}{N_c} \sum_{i=1}^{N_c} R_i^2 \quad (26)$$

$$R_i = (\hat{u}_t)_i + h((\hat{u})_i, (\hat{u}_x)_i, (\hat{u}_{xx})_i, \dots; \xi_i) \quad (27)$$

Training performance can deteriorate when the gradients associated with different loss components have substantially different magnitudes [35]. In the original Helmholtz example, the differential-equation residual dominates the training process, while the boundary-condition loss receives insufficient attention. Weighting the loss components provides a means of balancing their contributions during network training [35]. An adaptive weighting strategy is therefore employed to balance the contributions of the loss components. The weight estimates are calculated from gradient information and updated during training.

$$\hat{\lambda}_i = \frac{\max_{\theta n} \{|\nabla_{\theta} Loss_p(\theta n)|\}}{|\nabla_{\theta} \lambda_i Loss_i(\theta n)|}, i = 1, \dots, M \quad (28)$$

$$\begin{cases} \lambda_i^n = (1 - \alpha) \lambda_i^{n-1} + \alpha \hat{\lambda}_i, n > 1 \\ \lambda_i^1 = \hat{\lambda}_i, n = 1 \end{cases} \quad (29)$$

where $\alpha \in [0.05, 0.2]$.

The modified fully connected architecture and gradient-based loss weighting follow Wang et al. [34]. Two transformation networks encode the inputs, and element-wise interactions carry these features through the hidden layers. Here, these components are used with training-point enrichment near the current MPP and a decoupled RBDO procedure. The contribution of each component cannot be inferred from the performance of the complete framework alone.

- (1) $U = \phi(XW^1 + b^1), V = \phi(XW^2 + b^2)$
- (2) $H^{(1)} = \phi(XW^{z,1} + b^{z,1})$
- (3) $Z^{(k)} = \phi(H^{(k)}W^{z,k} + b^{z,k}), k = 1, \dots, L$
- (4) $H^{(k+1)} = (1 - Z^{(k)}) \square U + Z^{(k)} \square V, k = 1, \dots, L$
- (5) $f_{\theta}(x) = H^{(L+1)}W + b$

$$\theta = \{W^1, b^1, W^2, b^2, (W^{z,l}, b^{z,l})_{l=1}^L, W, b\} \quad (30)$$

Despite the incorporation of physical information, training a PINN for complex reliability problems can remain computationally demanding [36]. The quality of the training sample set is therefore important for improving model construction efficiency. For reliability analysis, the PINN is integrated into the ALPF-based FORM procedure. An initial training set is constructed from the boundary and differential-equation conditions, and the network is trained using the modified architecture and adaptive loss weights. The resulting response approximation is used to construct the optimization fitness function, from which the current MPP is obtained. Active learning concentrates subsequent training near the MPP; the original procedure evaluates the governing problem at the current MPP and adds the resulting information to the training set. The network and ALPF parameters are updated until the reliability estimate satisfies the prescribed convergence condition.

$$\frac{\|X^k - X^{k-5}\|}{\|X^k\|} \leq \varepsilon, k = 5, 10, \dots \quad (31)$$

The framework is further extended to RBDO with both random and interval variables. In this setting, interval uncertainty produces lower and upper bounds on the failure probability. The upper bound is compared with the target failure probability to assess whether a candidate design satisfies the reliability requirement.

$$\begin{aligned} & \text{find : } (d, \mu, Y) \\ & \text{min : } f(d, \mu_x, Y) \\ & \text{s.t. : } P_F \left(\min_{Y^L < Y < Y^U} g_i(d, X, Y) \leq 0 \right) - \Phi(-\beta_i^t) \leq 0, i = 1, 2, \dots, m \\ & \quad d^L \leq d \leq d^U, \mu_x^L \leq \mu_x \leq \mu_x^U, Y^L \leq Y \leq Y^U \\ & \quad P_F^L = P \left\{ \min_{Y^L < Y < Y^U} g(X, Y) \leq 0 \right\} \\ & \quad P_F^U = P \left\{ \min_{Y^L < Y < Y^U} g(X, Y) \leq 0 \right\} \end{aligned} \quad (32)$$

$$(33)$$

A decoupled solution strategy alternates between reliability assessment and design optimization. The minimum performance target point (MPTP) is determined for each constraint, and the associated shift vector is used to formulate the deterministic optimization subproblem. PINNs provide the responses required to evaluate the differential-equation-based constraints. The ALPF formulation and ESSO are then employed to solve the optimization problem.

$$\begin{cases} \min_{DV} g_i(U^{(k)}, Y) \\ \text{s.t. } \|U^{(k)}\| = \beta_i^t, (i = 1, 2, \dots, m, Y^L < Y < Y^U) \\ DV = \{x_i^{MPTP, (k)}, Y^{MPTP, (k)}\} \end{cases} \quad (34)$$

$$\begin{cases} s_{i,X}^{(0)} = 0, s_{i,Y}^{(0)} = 0 \\ s_{i,X}^{(k)} = \mu_x^{*(k-1)} - X_i^{MPTP, (k-1)} \\ s_{i,Y}^{(k)} = \bar{Y}^{(k-1)} - Y_i^{MPTP, (k-1)} \end{cases} \quad (35)$$

$$\begin{aligned} & \text{find : } (d, \mu_x, \bar{Y}) \\ & \text{min : } f(d, \mu_x, \bar{Y}) \\ & \text{s.t. : } g_i(d^{(k)}, \mu_x^{(k)} - s_{i,X}^{(k)}, \bar{Y}^{(k)} - s_{i,Y}^{(k)}) \geq 0, i = 1, 2, \dots, m \\ & \quad d^L \leq d^{(k)} \leq d^U, \mu_x^L \leq \mu_x^{(k)} \leq \mu_x^U, Y^L < \bar{Y}^{(k)} < Y^U \end{aligned} \quad (36)$$

The decoupled iterations continue until the design solution satisfies the convergence criterion.

$$\frac{\|d^{(iter)} - d^{(iter-1)}\|}{\|d^{(iter)}\| + 10^{-16}} \leq 10^{-3} \quad (37)$$

The riser example below examines the PINN-assisted design procedure with strength and deflection constraints. The numerical examples mentioned in the preceding method descriptions belong to the source studies and are not reproduced here. The present case therefore provides evidence for this particular implementation and response model, rather than validation across all methods and problem classes.

7. Engineering Application: Reliability-Based Design Optimization of a Deep-Water Riser

7.1. Problem description

The deep-water riser in Figure 1 provides a design problem in which the structural response must be obtained from differential equations [36, 37]. Its outer diameter and wall thickness are the two design variables. The aim is to reduce structural mass while meeting the reliability requirements for strength and lateral deflection. This example is used to assess the PINN-assisted optimization procedure.

The deep-water riser example is considered an idealized engineering benchmark for evaluating the proposed PINN-assisted reliability-based design optimization (RBDO) framework, rather than a comprehensive model of an operating offshore riser. A steady-current beam formulation is adopted to keep the structural response analysis computationally tractable and to focus on assessing the accuracy and computational efficiency of the proposed optimization methodology. The model retains the principal structural effects considered in the original formulation, including bending stiffness, effective axial tension, and lateral deflection under steady-current loading. However, it does not fully resolve the coupled dynamics of internal flow, ocean currents, waves, and vessel motion, or explicitly account for Coriolis effects, flow-induced buckling, and motion-dependent hydrodynamic and material damping. These simplifications define the scope of the present benchmark and do not imply that the omitted effects are negligible in practical offshore operations. Consequently, the results demonstrate the performance of the proposed methodology for the specified model and should not be interpreted as a comprehensive validation for fully coupled dynamic riser systems.

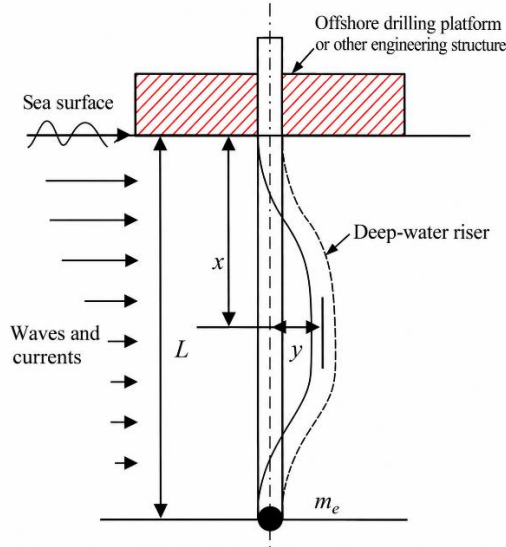


Figure 1. Structural configuration of the deep-water riser.

Changing the outer diameter or wall thickness changes the tubular cross-sectional area and hence the riser mass. The mass calculation also uses the steel density and water depth. The formulation below gives the objective function, the bounds on the two design variables, and the reliability constraints.

$$\begin{aligned}
 \min W(d) &= \rho_r \cdot A_r(D, t) \cdot L \\
 \text{s.t. } P_f(G_j(d, x, y) \leq 0) &\leq \Phi(-\beta_j^t), j = 1, 2 \\
 d &= [D, t]^T, d^L = [0.45, 0.012] \\
 d^U &= [0.55, 0.022], d^L \leq d \leq d^U \\
 \beta_1^t &= 3, \beta_2^t = 2.5
 \end{aligned} \tag{38}$$

The strength constraint compares the maximum normal stress at the top of the riser with the steel yield strength adjusted by the material partial factor. The stress calculation includes the effective axial tension and the other response terms in the structural model.

$$G_1 = \frac{\sigma_y}{\gamma M} - \sigma_{\max}(0) \quad (39)$$

$$\sigma_a(0) = \frac{T(0)}{Ar} \quad (40)$$

$$\sigma_b(0) = \frac{|M(0)|(D/2)}{I}$$

For the deflection constraint, the lateral displacement must remain below its allowable value. This displacement is calculated from the riser differential equation using the specified loads and boundary conditions.

$$G_2 = y_{allow} - |y(L)| \quad (41)$$

Table 1 lists four random variables: equivalent current velocity, steel yield strength, elastic modulus and wall-thickness manufacturing error. Yield strength is lognormally distributed; the other three variables are normally distributed. Two further quantities, the Morison drag coefficient and the equivalent concentrated mass at the lower end, are described by intervals. The problem therefore includes both random and interval uncertainty.

Table 1. Statistical properties and ranges of the uncertain variables.

Uncertainty type	Variable name	Symbol	Distribution type	Mean	Standard deviation	Range
Random variable	Equivalent current velocity (m/s)	U	Normal	0.60	0.06	[0.12,0.48]
	Steel yield strength (MPa)	σ_y	Lognormal	450	22.50	[382,517]
	Young's modulus (GPa)	E	Normal	206	6.18	[187.5,224.5]
	Wall thickness manufacturing error (m)	Δt	Normal	0	0.0417t	[-0.125t,0.125t]
Interval variable	Morison drag coefficient	C_D				[0.5,0.9]
	Equivalent lumped mass at the bottom end (t)	m_e				[180,220]

The response model assumes linear elastic behaviour and small lateral deflections. It includes bending stiffness, effective axial tension and a distributed drag load calculated from the prescribed equivalent current velocity. The boundary conditions are fixed during each response calculation. Internal-flow momentum coupling and wave- or vessel-induced transients are not resolved. The formulation contains no Coriolis term, and no separate criterion for flow-induced buckling is imposed. Hydrodynamic drag enters as a static load; motion-dependent hydrodynamic damping and material damping are outside this static formulation.

Pavlou investigated inner-flow-induced buckling in catenary risers [38], transient response in layered FRP shells [39], and coupled longitudinal-flexural-torsional motion in liquid-filled pipelines [40]. Pavlou and Ong examined damping in steel pipelines under fluid-hammer loading [41]. These studies concern different configurations and loading conditions, but identify dynamic and damping effects that the present static benchmark does not resolve. Their omission limits the results to the specified equilibrium problem and its two failure criteria. Use for operating risers would require an extended physical model and further validation.

The following equations define the lateral displacement, steady-current load, effective axial tension and boundary conditions for this restricted response model.

$$-\frac{d}{dx}\left(T(x)\frac{dy}{dx}\right)+EI\frac{d^4y}{dx^4}=f_c(x) \quad (42)$$

$$f_c(x)\approx\frac{1}{2}\rho_a C_D DU^2 \quad (43)$$

$$T(x)=\left[m_e+(\rho_r-\rho_a)\cdot A_r(L-x)\right]g \quad (44)$$

$$\begin{aligned} y(0) &= 0, y'(0) = 0 \\ y(L) &= 0, y'(L) = 0 \end{aligned} \quad (45)$$

7.2. Optimization results and discussion

Four solution approaches are compared: the conventional double-loop method, the decoupled method, the adaptive Kriging-assisted decoupled method (AK-DL), and the PINN-assisted decoupled method (PINN-DL). In the conventional double-loop and decoupled methods, the governing differential equation is solved using the bvp4c solver. PINN-DL evaluates the differential-equation-based responses through the trained physics-informed models within the decoupled optimization procedure. Table 2 presents the results reported in the original study.

Table 2. Comparison of optimization results for the deep-water riser.

Method	Optimal solution	Optimal objective value	Reliability indices of constraints	Computational time
Decoupled method	(0.4838, 0.0170)	293555.73	(2.9521, 2.5088)	40817.2426 s
Double-loop method	(0.4791, 0.0170)	280538.15	(2.9551, 2.4875)	250585.6048 s
AK-DL	(0.4740, 0.0181)	305252.31	(3.1455, 2.8126)	2782.0877 s
PINN-DL	(0.4813, 0.0170)	279406.20	(3.0002, 2.5005)	10867.5262 s

The double-loop method incurs the greatest computational expense because response calculations are repeatedly performed within the nested optimization and reliability-analysis procedures. The original study also reports that its resulting design violates the prescribed probability constraints and identifies an insufficient Monte Carlo sample size within the nested analysis as a possible cause. The conventional decoupled method reduces the computational time substantially relative to the double-loop approach, although repeated numerical solution of the governing equation remains costly.

AK-DL has the shortest computational time among the four methods. However, it produces the largest objective value, with a wall thickness of 0.0181 m. Its reported reliability indices are also higher than those obtained using PINN-DL. The original study identifies the AK-DL solution as not having reached the optimum in this example.

The PINN-DL design has an outer diameter of 0.4813 m and a wall thickness of 0.0170 m. Its objective value is 279406.20, with reliability indices of 3.0002 for strength and 2.5005 for deflection. Table 2 shows where this method gains and where it costs more: it takes less time than either conventional method and gives a lower objective value than AK-DL, but AK-DL finishes sooner. For this example, the additional computation used by PINN-DL produces the lighter of the two surrogate-assisted designs.

The PINN responses shown in Figure 2 agree with the corresponding bvp4c solutions. This comparison provides a numerical check on the response approximation used in the optimization. **Figure 2 also shows**

the evolution of the individual loss components during training. The data and physics losses decrease with similar overall trends, providing an additional indication of stable PINN training for this benchmark.

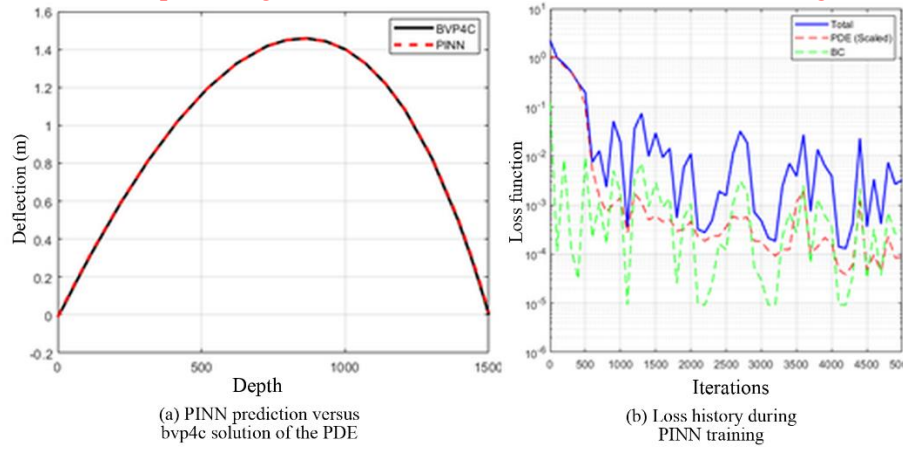


Figure 2. Comparison of PINN predictions with numerical solutions and evolution of the training-loss components.

In the riser example, the design obtained by the PINN-assisted decoupled procedure satisfies both reliability targets with random and interval uncertainties included in the calculation. Agreement with the bvp4c solutions supports the use of the trained network to calculate the riser response. Whether the procedure offers a computational saving depends on the method used for comparison, as shown by the timings in Table 2.

8. Conclusions

Much of the computation in the reliability and design problems considered here is spent evaluating structural responses. The methods in this paper address this expense at several stages of the calculation, from the MPP search to the construction and updating of the surrogate. They differ in the samples they require, the fidelity of the response evaluations and the information used in training. Any saving must be judged alongside the accuracy of the resulting reliability estimate.

The augmented Lagrangian formulation reduces dependence on a penalty factor fixed before the search, while ESSO makes the MPP optimization more robust. In the hybrid Kriging procedure, new samples are selected across the wider design space or around the current MPP, according to the needs of the update. Multi-fidelity Kriging adds a choice of model fidelity to the selection of each new sample. This allows computational savings through decisions about both where a response is evaluated and which model is used. The entropy-weighted ensemble deals with differences between surrogate predictions, combining information from the overall sample set with local prediction variance.

The PINN approach uses the governing differential equations and prescribed conditions in training the response model. Adaptive loss weights and the modified network architecture support this approximation, while active learning supplies further information near the MPP. The decoupled procedure then uses the trained models for design optimization with random and interval variables. In the riser comparison, this gives a lower objective value than AK-DL and shorter computation times than the conventional methods, but it does not give the shortest time overall.

The appropriate method depends on the information available for the problem. Multi-fidelity modelling requires a useful relationship between the cheaper and more expensive responses, whereas PINNs require physical equations and conditions that can be included in training. The present work mainly concerns static reliability calculations. Further work is needed to establish how these procedures perform for time-dependent responses and more complex operating conditions.

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