

A reduced-order 3D streamline simulation model for evaluating perforation-depth effects on injection energy in anisotropic reservoirs

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Abstract. High-fidelity reservoir simulators can predict well-pressure behaviour, but the mechanisms linking perforation depth, saturation changes, and injection-energy demand are often difficult to isolate. This paper presents an interpretable reduced-order 3D streamline model for water injection in anisotropic reservoirs with point-perforated vertical wells. It links perforation depth and 3D streamline geometry to well-pressure response and injection-energy consumption in an anisotropic reservoir. Traced streamlines are discretized into streamline-aligned segments to compute direction-dependent permeability, hydraulic resistance, flow redistribution, pressure drop, pumping power, cumulative energy, and production metrics. Buckley-Leverett transport along producer-connected streamlines estimates water breakthrough, [water cut](#), cumulative production, and energy consumed per barrel produced oil.

The model is applied to a North Sea-inspired configuration with a central producer and two flank injectors. The producer is perforated high in the reservoir, while injector perforation depth is varied from low to middle and high positions. The study evaluates effects on streamline geometry, well-pressure response, oil recovery and injection-energy efficiency, and assesses whether the oil-maximizing perforation strategy also minimizes energy per produced barrel. Selected benchmarks using OPM Flow, an open-source reservoir simulator, support the framework.

Keywords: streamline model, waterflooding, perforation depths, anisotropy, well bottom hole pressure (BHP) drop, pumping energy, OPM Flow.

1 Introduction

Water injection is widely used in offshore oil reservoirs to maintain reservoir pressure and improve hydrocarbon recovery. However, continuous water injection also requires pumping energy. For a given injection rate, this energy demand depends strongly on the pressure difference between the injector and producer wells. Therefore, well configurations that reduce the required pressure difference may improve both recovery and energy efficiency. [Previous studies have shown that treating and pumping water require significant energy, especially as water production increases over time \[1\].](#)

Perforation depth can be important in anisotropic reservoirs, where vertical permeability is often lower than horizontal permeability. Changing the vertical distance between injector and producer perforations changes the length, direction, and connectivity of the main flow paths. This can influence water breakthrough, hydraulic resistance, production, injection pressure, and pumping energy. The overall response depends not only on vertical permeability, but also on streamline geometry, cross-sectional area, fluid mobility, and redistribution of flow between different streamlines.

Full reservoir simulators can represent these processes in detail. However, it can be difficult to identify the main reason for a change in well pressure because several effects, such as saturation changes, near-well flow, mobility variation, and numerical treatment, occur at the same time. A simplified model can therefore complement full reservoir simulation by showing more clearly how reservoir flow geometry and fluid displacement affect pressure drop and energy demand.

[Streamline simulation provides a Lagrangian description of reservoir flow by following trajectories defined by the velocity field. Classical studies established streamline and streamtube methods for three-dimensional reservoir simulation \[2, 3, 4\], while Buckley–Leverett theory provides the basis for one-dimensional two-phase displacement along these flow paths \[5\]. Streamline methods are particularly useful for visualizing injector–producer connectivity and separating reservoir flow into individual transport paths.](#)

[More recent studies have continued to develop and apply streamline-based methods for waterflood analysis and reservoir management. Zhu et al. \[6\] combined streamline-derived interwell flux information with finite-volume reservoir simulation to evaluate injector–producer connectivity and improve waterflood management. Teixeira et al. \[7\] developed a streamline-based formulation for highly heterogeneous and anisotropic reservoirs using multipoint flux approximation and adaptive time stepping. Zhang et al. \[8\] presented a high-order streamline approach for waterflooding under prescribed pressure boundaries, while Kazem et al. \[9\] further evaluated streamline simulation for waterflood-management applications. These studies show that streamline information remains useful for efficient flow diagnosis, connectivity analysis, and waterflood screening.](#)

[Most such applications focus primarily on fluid displacement, sweep behaviour, injector–producer connectivity, or production management. In the present work, the emphasis is instead placed on linking three-dimensional streamline geometry and saturation-dependent hydraulic resistance directly to the pressure and injection-energy requirements associated with different perforation-depth configurations. The model is intended to help understand the main physical mechanisms and is not designed to replace a full reservoir simulator or predict absolute bottom-hole pressure for complex geological reservoirs.](#)

This paper addresses four questions:

1. How does injector–producer perforation-depth separation affect streamline geometry and hydraulic resistance?
2. How does saturation-dependent fluid mobility affect pressure drop, flow redistribution, and

pumping power?

3. Does the configuration that gives the highest oil production also give the lowest cumulative energy and energy per produced barrel?
4. Which configuration rankings and time-dependent trends are also reproduced by OPM Flow [10]?

The main contribution of this work is a reduced-order 3D streamline model that combines an analytical anisotropic point-well velocity field, frozen streamline geometry, Buckley–Leverett transport, resistance-based flow redistribution, near-well pressure correction, and production and energy calculations. OPM Flow is used as a benchmark for trends rather than as field validation. Section 2 presents the model formulation and computational workflow. Section 3 describes the case study and OPM Flow benchmarking strategy. Section 4 presents the results. [Section 5 discusses the physical interpretation, comparison with OPM Flow, research questions, model limitations, and possible extensions.](#) Section 6 summarizes the main conclusions.

2 Reduced-order 3D streamline model: formulation and workflow

2.1 Modelling concept and assumptions

The reservoir is assumed to be unbounded, homogeneous, and anisotropic. The principal permeabilities in the x , y , and z directions are denoted by K_x , K_y , and K_z , respectively, with vertical permeability lower than the horizontal permeabilities, $K_z < K_x, K_y$. Vertical wells are represented by point perforations in 3D Cartesian coordinates, and total injection equals total production. Streamlines are traced once from a steady single-phase velocity field and then kept fixed. Each traced streamline represents the centerline of a streamtube. Water saturation evolves along these fixed paths, while pressure drop changes come from saturation-dependent fluid mobility and resistance-based flow redistribution.

The model is grid-free, and all quantities are evaluated along the traced streamlines. Gravity, capillary pressure, compressibility, geological heterogeneity, changes in streamline geometry, and mechanical well constraints are neglected.

2.2 Analytical anisotropic velocity field and streamline tracing

Streamlines are traced from the analytical Darcy-velocity field obtained from the anisotropic pressure solution; the full derivation is given in Appendix (A). For multiple point wells, the contributions from injectors (sources) and producers (sinks) are superposed. For well i located at (x_{wi}, y_{wi}, z_{wi}) , define the anisotropic distance

$$R_i^2 = \frac{(x - x_{wi})^2}{K_x} + \frac{(y - y_{wi})^2}{K_y} + \frac{(z - z_{wi})^2}{K_z}.$$

The Darcy-velocity vector $\mathbf{u} = (u_x, u_y, u_z)$ is then

$$\mathbf{u}(x, y, z) = \frac{1}{4\pi\sqrt{K_x K_y K_z}} \sum_{i=1}^n \frac{Q_i (x - x_{wi}, y - y_{wi}, z - z_{wi})}{R_i^3}, \quad (1)$$

where n is the number of wells and Q_i is positive for injectors and negative for producers, with total injection equal to total production. For $K_x = K_y = K_z$, Equation (1) reduces to the isotropic point-well velocity field.

Streamlines are traced using the pore velocity $\mathbf{v} = \mathbf{u}/\phi$, where ϕ is porosity. To avoid the point-well singularity, tracing starts at an anisotropically scaled distance $r_{\text{start}} = 2$ m from each injector. The polar angle ψ is measured from the positive z -axis and the azimuthal angle θ in the x - y plane, as shown in Figure 1. Five polar angles sampled uniformly in $\cos \psi$ (the uniform sampling in $\cos \psi$ avoids excessive concentration of launch points near the poles), and 40 azimuthal angles $\theta \in [0, 2\pi)$ give 200 launch points per injector.

The corresponding launch coordinates are

$$\begin{aligned} x_0 &= x_w + a_x \sin \psi \cos \theta, \\ y_0 &= y_w + a_y \sin \psi \sin \theta, \\ z_0 &= z_w + a_z \cos \psi, \end{aligned} \quad a_\alpha = r_{\text{start}} \sqrt{\frac{K_\alpha}{K_{\text{ref}}}}, \quad K_{\text{ref}} = (K_x K_y K_z)^{1/3}, \quad (2)$$

where (x_w, y_w, z_w) is the injector location, $\alpha \in \{x, y, z\}$, and a_x , a_y , and a_z are the anisotropically scaled launch distances.

The streamline trajectory satisfies

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}(\mathbf{x}), \quad (3)$$

which is integrated using an adaptive fifth-order Runge–Kutta method (RK45) SciPy solver with relative and absolute tolerances of 10^{-7} and a maximum time step of 240 h. Tracing stops when a streamline enters the 2-m producer capture radius, escapes or stagnates, or reaches the 100-year tracing horizon. Paths reaching the producer are classified as captured; the remaining numerically truncated paths are treated as non-captured streamlines, as described in Subsection (2.5).

2.3 Frozen streamline geometry, area, and directional permeability

Each traced streamline is divided into 100 equal arc-length blocks of length l_b , where b denotes the block index. The initial reference flow rate assigned to each streamline from injector j is $q_0 = Q_j/N_j$, where Q_j is the water injection rate and N_j is the number of streamlines launched from that injector. From Darcy-flux conservation, the frozen cross-sectional area of block b is

$$A_b = \frac{q_0}{|\mathbf{u}_b|},$$

where $|\mathbf{u}_b|$ is the magnitude of the local Darcy-velocity vector evaluated at the midpoint of block b . Therefore, streamtube blocks are narrower near the wells, where velocity is higher, and wider in the far field.

For the unit tangent vector $\mathbf{t}_b = (t_{x,b}, t_{y,b}, t_{z,b})$ along block b , the effective directional permeability is

$$K_{\text{eff},b} = \left(\frac{t_{x,b}^2}{K_x} + \frac{t_{y,b}^2}{K_y} + \frac{t_{z,b}^2}{K_z} \right)^{-1}. \quad (4)$$

Thus, flow aligned with a principal direction gives the corresponding permeability (K_x , K_y , or K_z). For an isotropic reservoir, $K_x = K_y = K_z = K$, and Equation (4) gives $K_{\text{eff},b} = K$.

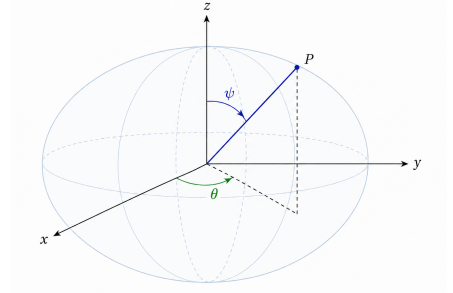


Figure 1. Launch angles ψ and θ .

2.4 Two-phase Buckley–Leverett transport model

Two-phase water displacement is solved independently along each traced streamline using the Buckley–Leverett equation [5],

$$A_i \phi \frac{\partial S_{w,i}}{\partial t} + q_i(t) \frac{\partial f_w(S_{w,i})}{\partial s} = 0, \quad (5)$$

where $S_{w,i}$ is the water saturation along streamline i , t is time, s is distance along the streamline, A_i is the local streamtube cross-sectional area, and $q_i(t)$ is the volumetric flow rate carried by streamline i . The flow rate varies over the simulation through resistance-based redistribution (see Equation (9) in Subsection (2.5)), but it is held fixed within each reporting interval and updated only at the reporting times. The water fractional flow is

$$f_w = \frac{\lambda_w}{\lambda_w + \lambda_o}, \quad \lambda_w = \frac{K_{rw}}{\mu_w}, \quad \lambda_o = \frac{K_{ro}}{\mu_o},$$

where λ_w and λ_o are the water and oil mobilities, and μ_w and μ_o are their viscosities. The relative permeabilities are described by Corey-type curves [11],

$$K_{rw} = K_{rw0} S_e^{n_w}, \quad K_{ro} = K_{ro0} (1 - S_e)^{n_o}, \quad S_e = \frac{S_w - S_{wc}}{1 - S_{wc} - S_{or}},$$

where S_{wc} and S_{or} are connate-water and residual-oil saturations, K_{rw0} and K_{ro0} are endpoint relative permeabilities of water and oil, and n_w and n_o are Corey exponents.

Each traced streamline is divided into 100 one-dimensional finite-volume blocks, as illustrated in Figure 2. The solid blue curve represents streamline i , and the dashed curves form its imaginary streamtube boundary; they are not physical reservoir boundaries or additional streamlines. The green lines mark the block faces. Block b has length $l_{i,b}$, area $A_{i,b}$, and cell-averaged saturation $S_{w,i,b}$. For positive $q_i(t)$, the arrows indicate fractional flow entering and leaving block b .

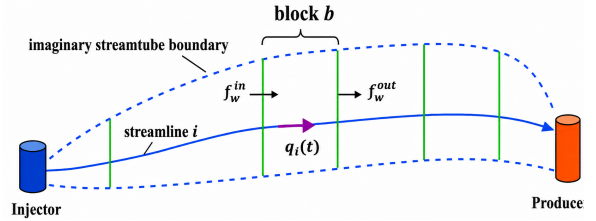


Figure 2. Finite-volume discretization around streamline i .

Within each reporting interval, the streamline rate q_i is held fixed, while the Buckley–Leverett equation is advanced using CFL-limited internal time steps. For streamline i ,

$$\Delta t_{\text{int}} \leq \text{CFL} \frac{\phi A_{i,b} l_{i,b}}{q_i \max |df_w/dS_w|}, \quad \text{CFL} = 0.89,$$

$$S_{w,i,b}^{n+1} = S_{w,i,b}^n + \frac{\Delta t_{\text{int}} q_i}{\phi A_{i,b} l_{i,b}} (f_w^{\text{in}} - f_w^{\text{out}}).$$

Here n and $n + 1$ denote successive internal transport time levels, with $t^{n+1} = t^n + \Delta t_{\text{int}}$. For positive q_i , the first-order upwind face values are $f_w^{\text{in}} = f_w(S_{w,i,b-1}^n)$ and $f_w^{\text{out}} = f_w(S_{w,i,b}^n)$. The Buckley–Leverett solution is advanced along every launched streamline over 4000 days. For captured streamlines, the final block represents the producer-side outlet, and for non-captured streamlines, it represents only the end of the known traced path and is used to update resistance, but not directly as the producer-side fluid composition.

2.5 Resistance, flow-rate redistribution and production metrics

The Buckley–Leverett saturation distribution changes the total fluid mobility and therefore the hydraulic resistance of each streamline. For streamline i , the block resistance and resistance over the available traced path are

$$r_{i,b}(t) = \frac{l_{i,b}}{K_{\text{eff},i,b} A_{i,b} \lambda_T(S_{w,i,b}(t))}, \quad \hat{R}_i(t) = \sum_b r_{i,b}(t), \text{ respectively,} \quad (6)$$

where $\lambda_T = \lambda_w + \lambda_o$. For a captured streamline, $\hat{R}_i$ represents the time-dependent resistance over complete traced injector–producer path; for a non-captured streamline it represents the resistance evolution only of the known traced portion.

Let $N_{c,j}$ be the total number of captured streamlines launched from injector j and C_j be the set containing them. Since the modelling choice is that all streamlines carry the same initial reference flow q_0 (defined in Subsection 2.3), we need to define the common reference resistance (baseline resistance) assigned to every streamline before any saturation or fluid mobility changes occur in that streamline. The initial resistance $\hat{R}_i(0)$ from Equation (6) of the traced path is used to define the following baseline resistance. [Its complete derivation](#) is given in Appendix (B).

$$R_{\text{base}} = \frac{N_{c,j}}{\sum_{i \in C_j} 1/\hat{R}_i(0)}. \quad (7)$$

Thus, R_{base} is the harmonic mean of the initial traced resistances of the captured streamlines from injector j . Hence the effective resistance evolution used in flow network of a streamline i is given as

$$R_i(t) = \underbrace{R_{\text{base}}}_{\text{baseline resistance}} + \underbrace{\left[\hat{R}_i(t) - \hat{R}_i(0) \right]}_{\text{change in resistance caused by evolving saturation/mobility}}. \quad (8)$$

For a captured streamline, the geometry of complete injector–producer path is known, so Equation (8) gives the resistance of entire producer-connected path. On the other hand, for a non-captured streamline, $\hat{R}_i(t)$ and $\hat{R}_i(0)$ cover only the traced portion, and the resistance of remaining untraced path is kept fixed at R_{base} over the time.

[Since fluid mobility keeps changing](#) over time along each traced streamline, the flow rate is redistributed among the traced paths. At each reporting time, the injector rate is redistributed across all streamlines from injector j according to

$$q_i(t) = Q_j \frac{1/R_i(t)}{\sum_{m \in I_j} 1/R_m(t)}, \quad \sum_{i \in I_j} q_i(t) = Q_j, \quad (9)$$

where I_j is the set of all streamlines launched from injector j .

For the cases considered here, let $\mathcal{I} = \bigcup_j I_j$ denote the set of all launched streamlines from all injectors. For a captured streamline, the assigned producer-side water saturation is $S_{w,i}^{\text{prod}}(t) = S_{w,i}^{\text{out}}(t)$, where $S_{w,i}^{\text{out}}$ is the saturation in its final producer-side block. For a non-captured streamline, $S_{w,i}^{\text{prod}} = S_{w,\text{init}}$, where $S_{w,\text{init}}$ is the initial reservoir water saturation, while the injected-water front remains within the known traced portion. Hence, the fraction of water in produced fluid is $f_{w,i}^{\text{prod}} = f_w(S_{w,i}^{\text{prod}})$, and the field water- and oil-production rates are

$$Q_w^{\text{prod}}(t) = \sum_{i \in \mathcal{I}} q_i(t) f_{w,i}^{\text{prod}}(t), \quad Q_o^{\text{prod}}(t) = \sum_{i \in \mathcal{I}} q_i(t) \left[1 - f_{w,i}^{\text{prod}}(t) \right]. \quad (10)$$

If the injected-water front reaches the final known block of a non-captured streamline, the calculation is stopped because its downstream composition is unknown. The cumulative water and oil production are

$$N_w(t) = \int_0^t Q_w^{\text{prod}}(\tau) d\tau, \quad N_o(t) = \int_0^t Q_o^{\text{prod}}(\tau) d\tau. \quad (11)$$

2.6 Well-to-well pressure drop, near-well pressure correction and energy metrics

The streamlines from each injector form a parallel hydraulic network. The equivalent resistance and reservoir-scale pressure drop for injector j are

$$R_{\text{eq},j}(t) = \left[\sum_{i \in I_j} \frac{1}{R_i(t)} \right]^{-1}, \quad \Delta p_{\text{res},j}(t) = Q_j R_{\text{eq},j}(t), \quad (12)$$

where I_j is the set of all streamlines launched from injector j .

Because streamline tracing starts and ends at a finite distance from the wells, $\Delta p_{\text{res},j}$ does not include the pressure loss between the streamline cutoff radius and the wellbore radius. This near-well contribution is estimated from radial Darcy flow. Peaceman [12, 13] developed the well-block pressure interpretation used in grid-based reservoir simulation to relate grid-block pressure to wellbore pressure. Here, a similar radial-flow relation is used between the streamline cutoff radius r_c and the wellbore radius r_w .

Near a vertical well, flow is approximated as radial in the horizontal plane. For a completion thickness h and radius r , the flow crosses a cylindrical surface of area $2\pi r h$. If q is the magnitude of the near-well flow rate, the radial Darcy velocity is

$$u_r = \frac{q}{2\pi r h}. \quad (13)$$

Neglecting capillary pressure, the two-phase Darcy flow can be written as

$$u_r = -K_h \lambda_{\text{nw}} \frac{dp}{dr}, \quad (14)$$

where λ_{nw} is the local total mobility and $K_h = \sqrt{K_x K_y}$ is the effective horizontal permeability for radial flow around a vertical well. Combining equations (13) and (14) gives

$$\frac{dr}{r} = -\frac{2\pi h K_h \lambda_{\text{nw}}}{q} dp. \quad (15)$$

Integrating between r_w and r_c gives the following radial-flow pressure loss magnitude

$$\Delta p_{\text{rad}} = \frac{q}{2\pi K_h h \lambda_{\text{nw}}} \ln \left(\frac{r_c}{r_w} \right).$$

To account for additional near-well effects, a dimensionless skin factor S is added to the logarithmic term, giving

$$\Delta p_{\text{nw}} = \frac{q}{2\pi K_h h \lambda_{\text{nw}}} \left[\ln \left(\frac{r_c}{r_w} \right) + S \right], \quad (16)$$

where Δp_{nw} is taken as the positive pressure-loss magnitude. A positive S represents additional near-well resistance, while a negative S represents a reduction relative to the ideal radial-flow case. A complete physical and mathematical interpretation of skin factor S is given in Appendix

(C). At the injector, $q = Q_j$ and $\lambda_{\text{nw}} = \lambda_T(S_{w,\text{inj}})$, where $S_{w,\text{inj}}$ is the injected-water saturation. At the producer, the same assigned producer-side saturations $S_{w,i}^{\text{prod}}$ defined in Subsection 2.5 are used to calculate the following flow-weighted effective mobility,

$$\lambda_{\text{prod}}^{\text{eff}}(t) = \frac{\sum_{i \in \mathcal{I}} q_i(t) \lambda_T(S_{w,i}^{\text{prod}}(t))}{\sum_{i \in \mathcal{I}} q_i(t)}. \quad (17)$$

The total producer flow and $\lambda_{\text{prod}}^{\text{eff}}$ are used in Equation (16) to calculate the producer near-well pressure loss. The corrected injector–producer pressure drop is

$$\Delta p_j(t) = \Delta p_{\text{res},j}(t) + \Delta p_{\text{nw},\text{inj},j}(t) + \Delta p_{\text{nw},\text{prod}}(t), \quad (18)$$

where $\Delta p_{\text{nw},\text{inj},j}$ and $\Delta p_{\text{nw},\text{prod}}$ are the injector- and producer-side near-well pressure losses, respectively, and $\Delta p_j \simeq p_{\text{BHP},\text{inj},j} - p_{\text{BHP},\text{prod}}$.

For pump efficiency η_p , the injector and total pumping powers are

$$P_j(t) = \frac{Q_j \Delta p_j(t)}{\eta_p}, \quad P_{\text{tot}}(t) = \sum_j P_j(t). \quad (19)$$

Cumulative energy and energy per produced oil are

$$E(t) = \int_0^t P_{\text{tot}}(\tau) d\tau, \quad e_o(t) = \frac{E(t)}{N_o(t)}. \quad (20)$$

The energy intensity is reported in kWh m⁻³ and kWh bbl⁻¹, using 1 m³ = 6.28981 bbl.

2.7 Computational workflow

Figure 3 summarizes the complete modelling workflow. Reservoir, fluid, and well properties are first used to calculate the analytical velocity field and trace the streamlines. The traced paths are then fixed and discretized into streamtube blocks. Buckley–Leverett transport updates water saturation, after which streamline resistance and flow rates are recalculated. Production, pressure drop, and energy metrics are evaluated at each reporting time, and the updated streamline flow rates are used in the next transport interval.

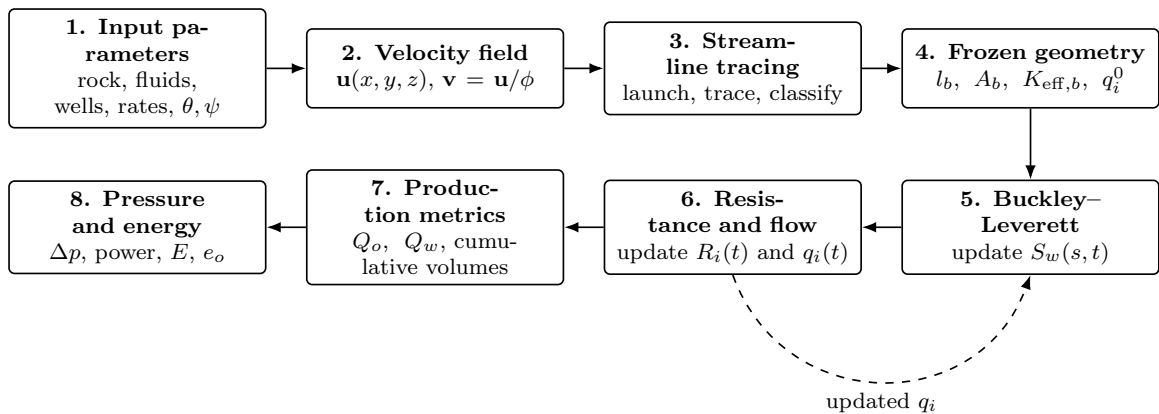


Figure 3. Computational workflow of the reduced-order 3D streamline model.

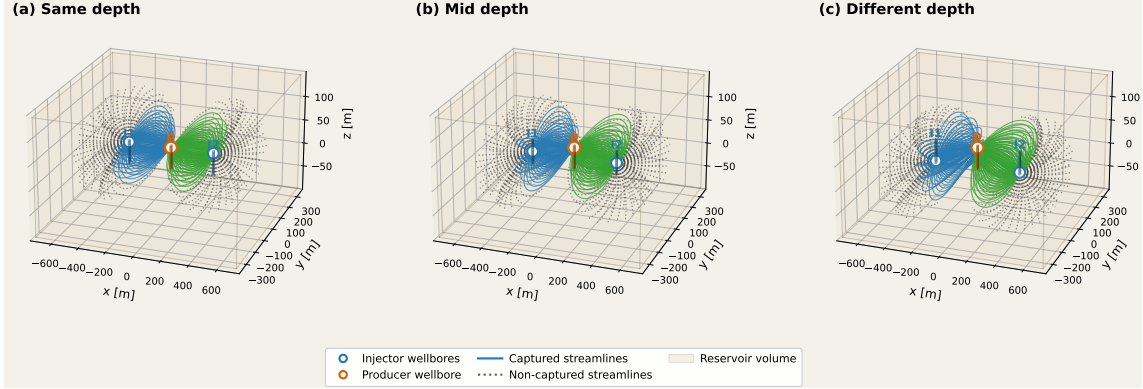


Figure 4. Three-dimensional streamline configurations for the (a) same-depth, (b) mid-depth, and (c) different-depth cases. Solid colored lines denote captured streamlines connecting the injectors to the producer, while dotted gray lines denote non-captured streamlines. The wellbores and reservoir volume are shown schematically, and the vertical scale is visually emphasized for clarity.

Table 1. Common reservoir, fluid, and model parameters.

Parameter	Value	Parameter	Value
ϕ	0.30	$K_x = K_y, K_z$	100, 10 mD
μ_w, μ_o	1.0, 1.0 cP	$S_{wc} = S_{w,init}, S_{or}, S_{w,inj}$	0.10, 0.30, 0.70
K_{rw0}, K_{ro0}	0.80, 1.00	n_w, n_o	2.0, 2.5
r_c, r_w	2, 0.125 m	Completion thickness	11 m
Simulation time	4000 days	η_p	0.75

3 Case study, verification, and benchmarking

An idealized symmetric pattern with one central producer and two flank injectors is used to examine the effect of injector–producer vertical separation. Rock and fluid properties, horizontal well spacing, flow rates, and numerical settings are fixed, while only the injector depth is varied. The 3D streamline model is evaluated through a single-streamline comparison with 1D OPM Flow and a three-dimensional benchmark against OPM Flow.

3.1 Well geometry and perforation-depth cases

The producer is located at (0, 0, 40) m and the two injectors at ($\pm 315, 0, z_I$) m. Injector elevations $z_I = 40, 20$, and 0 m define the same-, mid-, and different-depth cases, corresponding to vertical separations $\Delta z = 0, 20$ and 40 m, respectively. Each injector supplies $25 \text{ m}^3 \text{ h}^{-1}$, while the producer rate is $50 \text{ m}^3 \text{ h}^{-1}$.

For each injector, streamlines are launched over $N_\theta = 40$ azimuthal angles and $N_\psi = 5$ polar angles, giving 200 launch directions per injector and 400 streamlines per case. Figure 4 shows the captured and non-captured streamline configurations for the three perforation-depth cases and 100 years of tracing horizon, while the common model parameters are summarized in Table 1.

3.2 OPM Flow verification and benchmarking

Verification and benchmarking are performed at two levels. A one-dimensional OPM model first verifies the hydraulic resistance, saturation-dependent mobility, and injector-producer pressure

drop along a single streamline. The complete 3D streamline model is then benchmarked against matched OPM waterflood simulations for the three perforation-depth cases.

Single-streamline verification. To verify the injector–producer pressure-drop calculation independently of the full 3D streamline network, one horizontal streamline is traced from an injector at $(-315, 0, 40)$ m to the producer at $(0, 0, 40)$ m using the adaptive RK45 solver, with one polar angle, $\psi = \pi/2$, and one azimuthal angle, $\theta = 0$. For this verification, the streamline is assigned a flow rate of $9 \text{ m}^3 \text{ h}^{-1}$ ($216 \text{ m}^3 \text{ d}^{-1}$). The traced reservoir segment has a length of 310.0644 m and is discretized into 100 equal arc-length blocks, giving $l_b \approx 3.1006$ m. The maximum streamtube cross-sectional area is 49280.99 m^2 .

The blockwise streamline geometry is mapped to an equivalent $100 \times 1 \times 1$ Cartesian OPM Flow model. A constant OPM reference area is defined as

$$A_{\text{ref}} = \max_b A_b = 49280.99 \text{ m}^2,$$

where A_b is the cross-sectional area of streamline block b . The transverse cell dimensions are therefore $\Delta y^{\text{OPM}} = \Delta z^{\text{OPM}} = \sqrt{A_{\text{ref}}} = 221.9932$ m. The 98 interior OPM cells have $\Delta x_b^{\text{OPM}} = 3.1006$ m, while the first and last cell lengths are doubled to 6.2012 m. Their porosities are correspondingly halved so that the pressure difference between the endpoint cell centres represents the full traced streamline length without changing the endpoint pore volumes.

Each streamline block is mapped to one corresponding OPM cell. The equivalent OPM permeability K_b^{OPM} and porosity ϕ_b^{OPM} are assigned from

$$K_b^{\text{OPM}} A_{\text{ref}} = K_{\text{eff},b} A_b, \quad \phi_b^{\text{OPM}} A_{\text{ref}} \Delta x_b^{\text{OPM}} = \phi A_b l_b, \quad (21)$$

where $K_{\text{eff},b}$, A_b , l_b , and ϕ are the directional permeability, cross-sectional area, length, and porosity of streamline block b , respectively. The permeability mapping preserves the hydraulic conductance associated with permeability and cross-sectional area, while the porosity mapping preserves the corresponding block pore volume. The same mapped permeability is assigned to the three permeability directions of each OPM cell because the equivalent model contains only one flow path in the x direction.

The fluid properties, relative-permeability representation, saturation conditions, flow rate, and simulation time are matched between the two representations. For the endpoint well-connection calculations, the completion thickness is set to the OPM cell thickness, $h = \Delta z^{\text{OPM}} = 221.9932$ m. The injector- and producer-side connection permeabilities are taken from the first and last equivalent OPM cells, giving $K_{\text{inj}} = 0.1622344$ mD and $K_{\text{prod}} = 0.1016271$ mD, respectively. The reservoir-only pressure variation calculated by the streamline model from Equation (12) is compared directly with the OPM endpoint-cell pressure difference, $p_{\text{BPR},1} - p_{\text{BPR},100}$.

The analytical injector- and producer-side near-well losses are calculated from Equation (16) using $r_c = 2$ m, $r_w = 0.125$ m, and an effective connection skin factor of $S = 2.74$. The injector-side mobility is evaluated at the imposed injected-water saturation, whereas the producer-side total mobility is evaluated from the saturation of the final streamline block. The corrected pressure difference from Equation (18) is then compared with the OPM well-to-well BHP difference, $p_{\text{BHP},\text{INJ}} - p_{\text{BHP},\text{PROD}}$.

Complete 3D streamline model benchmark. The complete 3D streamline model is benchmarked against three matched OPM Flow waterflood cases representing the same-, mid-, and different-depth configurations. The OPM model uses a $40 \times 40 \times 140$ Cartesian grid with cell dimensions

$45 \times 27.5 \times 1$ m. It is initialized at 500 bar pressure and water saturation $S_{wc} = 0.10$ and uses the same rock, fluid, and relative-permeability properties as given in Table 1. Equal oil and water densities of 999.9 kg m^{-3} are specified to suppress gravity segregation.

The producer is located at grid position (20, 20) and the injectors at (13, 20) and (27, 20), giving a horizontal injector–producer spacing of 315 m. Each well is completed over 11 layers. The producer occupies layers 40–50, while the injector intervals are 40–50, 60–70, and 80–90 for the same-, mid-, and different-depth cases, respectively. The corresponding completion-centre separations are therefore 0, 20, and 40 m, matching the streamline-model geometries.

Each injector is water-rate controlled at $600 \text{ m}^3 \text{ d}^{-1}$ ($25 \text{ m}^3 \text{ h}^{-1}$), while the producer is liquid-rate controlled at $1200 \text{ m}^3 \text{ d}^{-1}$ ($50 \text{ m}^3 \text{ h}^{-1}$). All cases are simulated for 4000 days with 10-day reporting intervals. The OPM well-to-well pressure difference is calculated as

$$\Delta p_{\text{OPM}}(t) = \frac{p_{\text{BHP},I1}(t) + p_{\text{BHP},I2}(t)}{2} - p_{\text{BHP},P}(t), \quad (22)$$

where t is reporting time, $p_{\text{BHP},I1}$ and $p_{\text{BHP},I2}$ are the bottom-hole pressures of injector $I1$ and $I2$, respectively, and $p_{\text{BHP},P}$ is the producer bottom-hole pressure. This OPM Flow quantity is compared with the corresponding 3D streamline model’s injector-averaged corrected pressure drop from Equation (18), which combines the reservoir contribution from Equation (12) with the injector- and producer-side near-well pressure losses from Equation (16) with skin factor value $S = -2$. Water cut, and oil- and water-production responses are also compared. Pumping power, cumulative injection energy, and energy per produced oil are then evaluated from Equations (19) and (20) using the respective pressure drop histories. The comparison is used as a system-level benchmark rather than an exact numerical equivalence test.

4 Results

Results are presented following the two-level verification and benchmarking strategy described in Subsection 3.2.

4.1 Single-streamline versus 1D OPM Flow

Figure 5 compares the single-streamline pressure-drop response with its equivalent 1D OPM model, as described in Subsection 3.2. Figure 5(a) shows that the reservoir-only pressure variations agree closely over the full simulation. At 4000 days, OPM gives 53.458 bar and the streamline model 53.672 bar, a difference of only 0.214 bar (0.40%).

For the well-to-well BHP comparison, the analytical near-well correction is applied with the effective connection skin factor $S = 2.74$ (Figure 5(b)). The models differ during the initial OPM pressure adjustment but rapidly converge. After about 100 days, the BHP curves remain nearly coincident. From 100 to 4000 days, the mean absolute difference is 0.125 bar (0.0069% relative), with a maximum difference of 0.375 bar. The reservoir-only agreement (Figure 5(a)) verifies the streamline calculation of pressure loss through the porous flow path, while the corrected BHP comparison (Figure 5(b)) shows that the analytical near-well formulation reproduces the corresponding OPM BHP response closely.

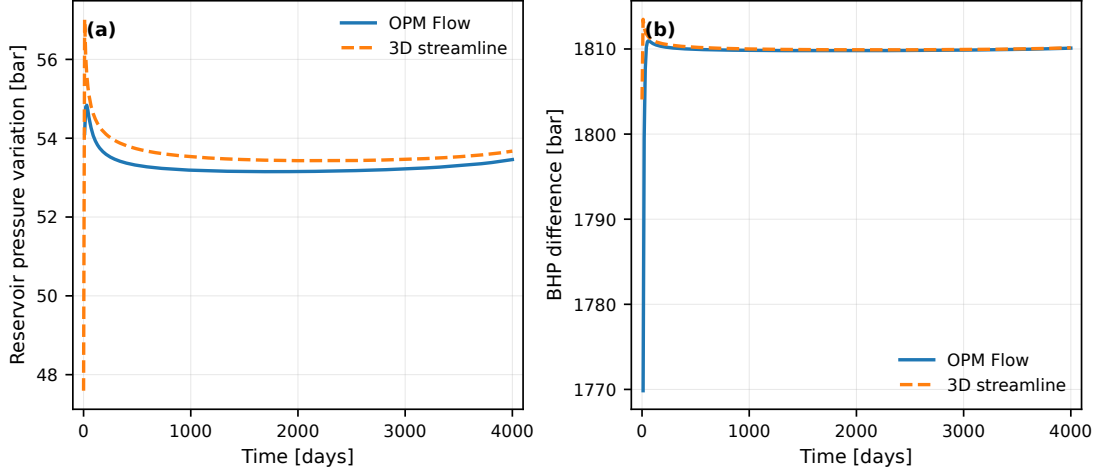


Figure 5. Single-streamline verification against the equivalent 1D OPM Flow model: (a) reservoir-only pressure variation and (b) injector-to-producer BHP difference.

4.2 Complete 3D streamline model versus OPM Flow

Pressure drop and pumping power. Figure 6 compares the injector–producer pressure drop for the three perforation-depth cases. OPM shows an initial pressure decline followed by a smooth increase, whereas the 3D streamline model gives a more stepwise response as individual streamlines undergo saturation and resistance changes. Both models nevertheless predict the same late-time ordering, $\Delta p_{\text{same}} > \Delta p_{\text{mid}} > \Delta p_{\text{different}}$.

At 4000 days, the streamline model underpredicts the OPM pressure drop by approximately 14.6–15.5% (Table 2), while preserving the same effect of perforation-depth separation. From the same- to different-depth case, the final pressure drop decreases by 9.76% in OPM and 8.76% in the streamline model. Because the injection rates and pump efficiency are fixed, pumping power follows the pressure-drop response directly through Equation (19). At 4000 days, both models identify the different-depth case as requiring the lowest pumping power, as summarized in Table 2.

Water breakthrough and production response. Figure 7 compares water cut and cumulative production. Using a water cut threshold of 1%, OPM predicts breakthrough at 1440, 1610, and 2050 days for the same-, mid-, and different-depth cases, whereas the streamline model predicts 1330, 1360, and 1680 days, respectively. Thus, the streamline model predicts earlier water arrival, but both models show that increasing vertical separation delays breakthrough.

At 4000 days, the streamline model predicts lower final water cut than OPM for all three cases, while both models identify the different-depth case as producing the least water and the most oil. The final cumulative oil deviations from OPM are approximately +0.19%, −0.93%, and −1.12% for the same-, mid-, and different-depth cases, respectively. The final production quantities are summarized in Table 2.

Figure 8 shows the corresponding production-rate histories. Total injection and total produced-liquid rates remain at $1200 \text{ m}^3 \text{ d}^{-1}$, while oil production decreases and water production increases after breakthrough. The streamline rates are more stepwise than the OPM responses, but the delayed water production with increasing perforation-depth separation is reproduced consistently.

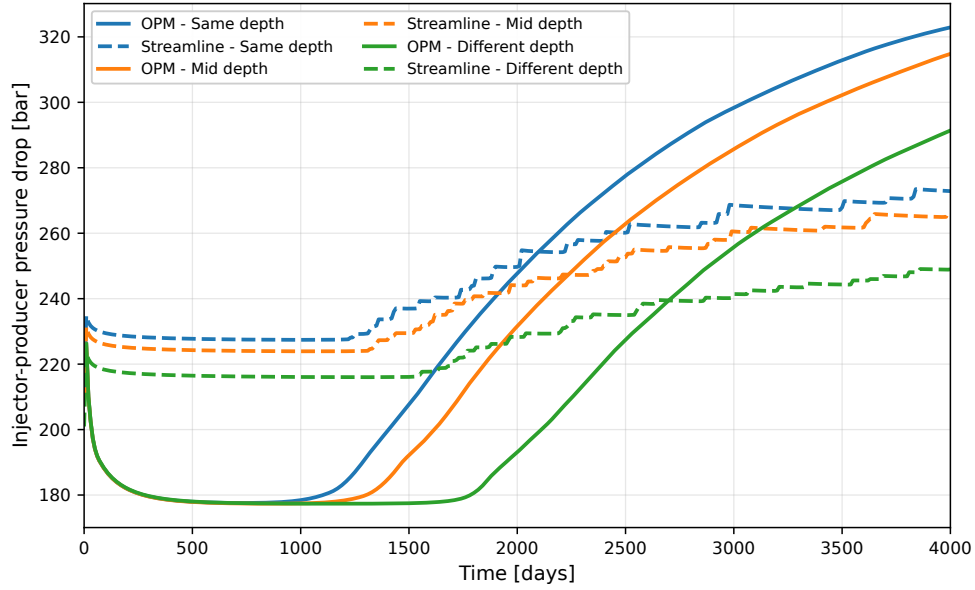


Figure 6. Injector-producer pressure-drop comparison between OPM Flow and the 3D streamline model for the same-, mid-, and different-depth cases.

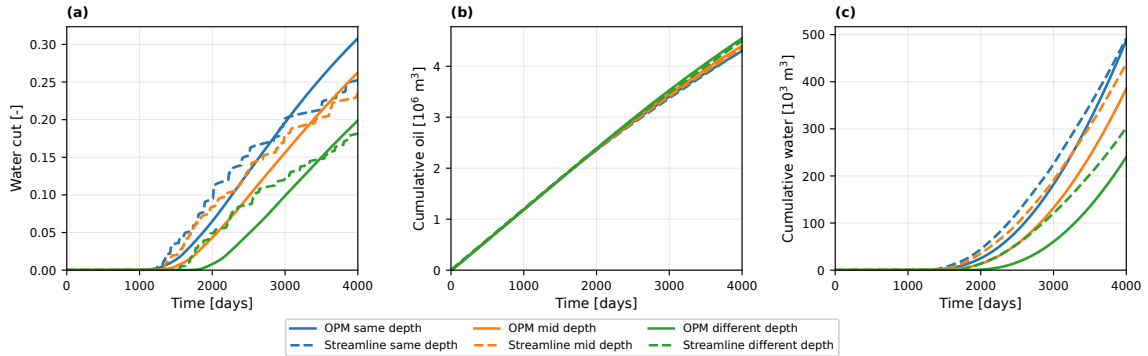


Figure 7. Comparison of OPM Flow and 3D streamline-model production responses for the three perforation-depth cases: (a) water cut, (b) cumulative oil production, and (c) cumulative water production.

Cumulative energy and energy per produced oil. Figure 9 compares cumulative injection energy and energy per produced oil. At 4000 days, the streamline model predicts slightly higher cumulative energy than OPM, with differences of approximately 1.9%, 3.5%, and 6.7% for the same-, mid-, and different-depth cases, respectively.

Both models nevertheless give the same energy-efficiency ranking, with energy per produced oil decreasing from the same- to the different-depth case. Relative to the same-depth configuration, the different-depth case reduces the final energy per barrel by 16.8% in OPM and 11.7% in the streamline model. Final energy values are summarized in Table 2.

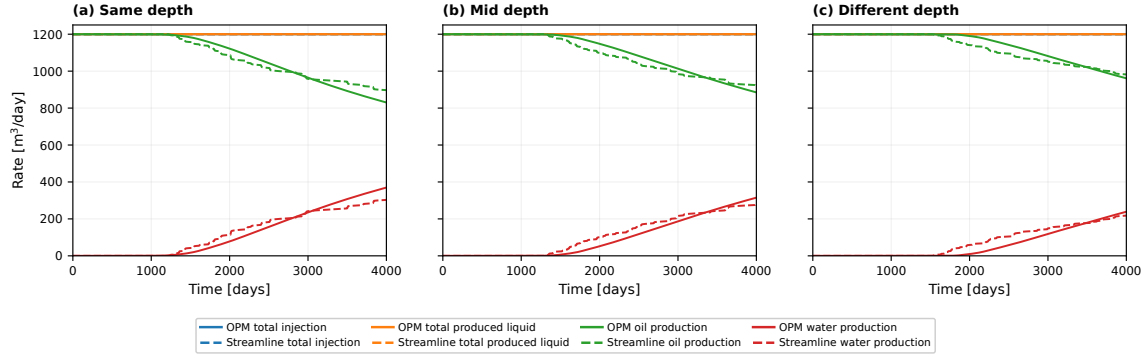


Figure 8. Comparison of OPM Flow and 3D streamline-model injection and production rates for the (a) same-depth, (b) mid-depth, and (c) different-depth cases.

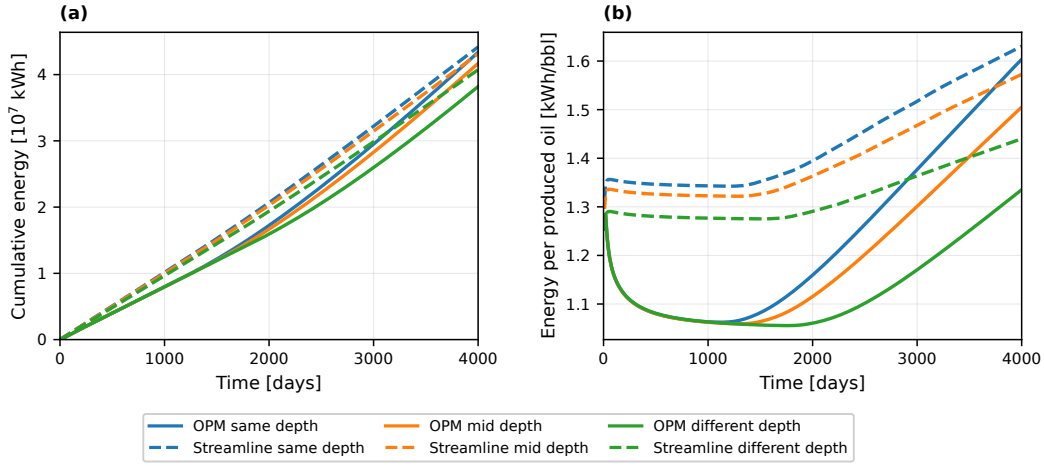


Figure 9. Comparison of OPM Flow and 3D streamline-model energy responses: (a) cumulative injection energy and (b) energy per produced oil.

Table 2. Summary of OPM Flow and 3D streamline-model results. Final values correspond to 4000 days. Water breakthrough is defined as the first time at which the producer water cut reaches 1% ($WC = 0.01$).

Model	Case	t_{bt} [d]	Δp_f [bar]	$P_{tot,f}$ [kW]	WC_f [-]	$N_{o,f}$ [10^6 m ³]	$N_{w,f}$ [10^3 m ³]	E_f [10^7 kWh]	$e_{o,f}$ [kWh/bbl]
OPM	Same	1440	322.90	597.96	0.308	4.300	487.7	4.337	1.604
OPM	Mid	1610	314.80	582.97	0.263	4.403	385.3	4.168	1.505
OPM	Different	2050	291.39	539.62	0.199	4.548	240.0	3.819	1.335
3D streamline	Same	1330	272.87	505.31	0.252	4.308	492.2	4.418	1.630
3D streamline	Mid	1360	266.41	493.35	0.235	4.362	438.3	4.314	1.573
3D streamline	Different	1680	248.96	461.04	0.182	4.497	302.5	4.074	1.440

t_{bt} : water-breakthrough time; Δp_f : final injector–producer pressure drop; $P_{tot,f}$: final total pumping power; WC_f : final producer water cut; $N_{o,f}$ and $N_{w,f}$: final cumulative oil and water production; E_f : final cumulative injection energy; and $e_{o,f}$: final energy per barrel produced oil.

Table 3. Selected streamline-geometry diagnostics.

Case	N_{cap} [—]	F_{cap} [—]	$\bar{L}_{\text{cap}}$ [m]	$\bar{A}_{\text{cap}}$ [m ²]	$ \mathbf{u} _{\text{cap}}$ [m/h]
Same	230	0.575	394.7	202.0	1.53×10^{-2}
Mid	230	0.575	397.2	216.5	1.46×10^{-2}
Different	234	0.585	411.9	261.3	1.30×10^{-2}

N_{cap} : captured streamlines; F_{cap} : initial captured-flow fraction; $\bar{L}_{\text{cap}}$: mean captured streamline length; $\bar{A}_{\text{cap}}$: mean captured streamline block area; $|\mathbf{u}|_{\text{cap}}$: mean Darcy velocity.

5 Discussion

The single-streamline comparison in Subsection 4.1 provides a controlled verification of the hydraulic formulation before the complete 3D network is considered. The reservoir-only pressure variations agree closely, with a difference of only 0.40% at 4000 days. This shows that the streamline block geometry, directional permeability, saturation-dependent mobility, and series-resistance calculation can reproduce the pressure loss of an equivalent 1D finite-volume model. The porous-path resistance calculation is therefore strongly supported by the equivalent 1D comparison. The near-well-corrected BHP response also agrees closely with the equivalent 1D OPM model after the initial pressure adjustment, supporting the analytical treatment of the well-connection losses.

The complete 3D benchmark in Subsection 4.2 shows how perforation-depth separation influences pressure drop, water breakthrough, production, and injection-energy demand. Figure 10 shows that the main geometric changes occur in the x - z plane, while the corresponding x - y projections remain comparatively similar. As the injectors are placed farther below the producer, the producer-connected streamlines spread over a wider vertical region rather than remaining concentrated directly toward the producer. Table 3 shows that the number of captured streamlines and the initial captured-flow fraction remain similar across the three cases, with a slight increase in the different-depth case. The mean captured streamline length increases from 394.7 to 411.9 m, while the mean streamtube area increases from 202.0 to 261.3 m² and the mean Darcy velocity decreases from 1.53×10^{-2} to 1.30×10^{-2} m/h. The increase in streamtube area is consistent with $A_b = q_0/|\mathbf{u}_b|$ in Subsection 2.3.

Equation (6) shows that resistance is directly proportional to streamline length and inversely proportional to streamtube area, directional permeability, and total mobility. The different-depth case therefore does not achieve a lower pressure drop through shorter flow paths; its mean captured streamline length is in fact larger. Instead, the results indicate that the broader vertical distribution of the producer-connected flow paths and the larger streamtube areas reduce the hydraulic resistance sufficiently to outweigh the longer flow paths and the additional contribution of vertical flow through the lower vertical permeability, $K_z < K_x = K_y$.

The same geometric changes are consistent with the observed production and energy trends. Greater vertical separation between injector and producer perforations produces a wider vertical distribution of the connected flow paths and is accompanied, in the cases investigated, by delayed water breakthrough, lower final water cut, and higher cumulative oil production. The different-depth case consequently gives the highest cumulative oil production and the lowest cumulative water production in both models. At the fixed injection rate, its lower pressure drop also reduces pumping power and cumulative injection energy. Combined with the higher oil production, this gives the lowest energy consumed per produced barrel. Thus, for the present case, the configuration with the highest oil production is also the most energy-efficient.

The OPM benchmark supports these conclusions at the trend level. Both models give the

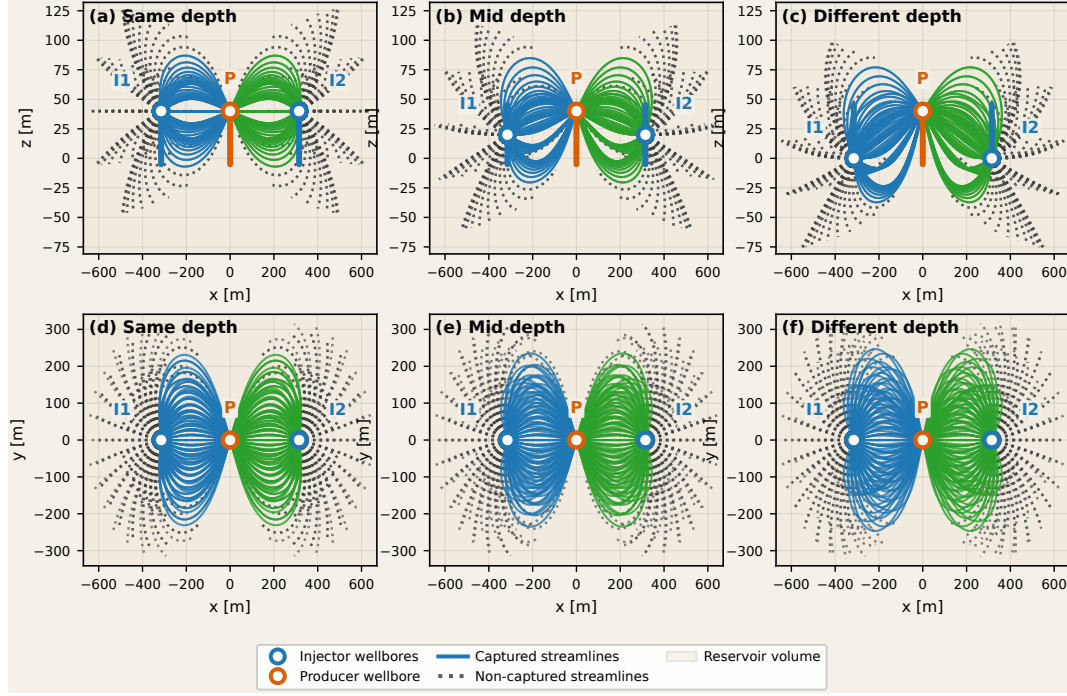


Figure 10. Projected streamline geometry for the same-, mid-, and different-depth cases: (a)–(c) x - z projections and (d)–(f) corresponding x - y projections. These are projection views of the same traced streamline cases shown in Figure 4.

same overall ranking of the three perforation-depth configurations, with the different-depth case giving lower pressure and energy demand together with improved production performance. Differences remain in absolute magnitudes and timing. The 3D streamline model predicts earlier water breakthrough, lower late-time pressure drop, and more stepwise pressure and production responses. Its cumulative injection energy at 4000 days is slightly higher than OPM, with the difference increasing from about 1.9% in the same-depth case to 6.7% in the different-depth case. Although the reservoir properties, well spacing, perforation-depth separation, and flow rates are matched, the spatial representations are different. OPM Flow solves the flow equations on a finite Cartesian grid in a bounded homogeneous anisotropic reservoir and represents the wells through completed grid cells. In contrast, the 3D streamline model uses analytical point-well velocities, adaptively traced streamlines, and frozen streamtube geometries in an unbounded homogeneous anisotropic domain. Exact quantitative agreement is therefore not expected.

Overall, the results answer the four research questions posed in Section 1. Increasing perforation-depth separation mainly changes the vertical flow geometry and lowers the hydraulic resistance of the producer-connected network. Saturation-dependent mobility changes the resistance of individual flow paths and drives time-dependent flow redistribution, pressure, and power responses. For the present anisotropic case, the configuration with the highest oil production also gives the lowest cumulative energy and energy per produced barrel, while OPM Flow reproduces the same principal configuration rankings and trends.

The study remains idealized. The reservoir is homogeneous, the streamtube geometry is frozen, and gravity, capillary pressure, compressibility, and geological heterogeneity are neglected. Wells are also represented by point perforations in the streamline model and by finite completion intervals in OPM. These assumptions limit quantitative predictive accuracy but

help isolate the mechanisms examined here. Future work should investigate broader ranges of anisotropy, well spacing, mobility ratio, and relative-permeability functions, together with the effects of gravity and reservoir heterogeneity.

6 Conclusions

This study developed an interpretable 3D streamline model to investigate how injector–producer perforation-depth separation affects pressure response, water displacement, production, and injection-energy demand in an anisotropic reservoir. The model combines analytical velocity fields, adaptive RK45 streamline tracing, streamline-aligned Buckley–Leverett transport, directional permeability, hydraulic resistance, flow redistribution, near-well pressure correction, and energy calculations within a common framework.

The single-streamline verification showed close agreement with the equivalent 1D OPM Flow model, with only a 0.40% difference in reservoir pressure drop at 4000 days. The complete 3D model reproduced the main perforation-depth trends observed in OPM Flow. For the cases investigated, increasing vertical separation was associated with delayed water breakthrough, lower pressure drop and pumping demand, lower water production, and higher cumulative oil production. Consequently, the different-depth configuration gave both the highest oil production and the lowest energy per produced barrel. The streamline diagnostics indicate that these trends are associated with a wider vertical distribution of the connected flow paths, larger streamtube areas, lower mean Darcy velocity, and the resulting changes in hydraulic resistance. The model is therefore intended to complement, rather than replace, full reservoir simulation by providing a transparent framework for interpreting and screening well configurations in terms of both production and injection-energy performance.

Acknowledgments The authors are grateful to Leading Researcher Dag Chun Standnes at Equinor ASA Bergen, Norway, for his constructive suggestions and technical guidance in preparing and troubleshooting the OPM Flow input decks, which improved the results of this paper.

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Appendices

A Derivation of the analytical anisotropic point-well velocity field

According to Darcy's law, the Darcy velocity is governed by the pressure gradient. Consider single-phase flow in a homogeneous porous medium with constant porosity and incompressible rock and fluid. Mass conservation is

$$\frac{\partial(\phi\rho)}{\partial t} + \nabla \cdot (\rho\mathbf{u}) = \rho q, \quad (23)$$

where ϕ is porosity, ρ is fluid density, $\mathbf{u} = (u_x, u_y, u_z)$ is the Darcy velocity, and q is the volumetric source or sink rate per unit bulk volume. Positive q denotes injection and negative q production. For constant ϕ and incompressible fluid, $\partial(\phi\rho)/\partial t = 0$. For a point well, the source or sink is concentrated at the well location; hence, away from this singular point, $q = 0$ and

$$\nabla\mathbf{u} = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} = 0. \quad (24)$$

The well strength is introduced later through the total flux across a closed surface surrounding the well. Darcy's law is

$$\mathbf{u} = -\frac{\mathbf{K}}{\mu}\nabla P, \quad \mathbf{K} = \begin{bmatrix} K_x & 0 & 0 \\ 0 & K_y & 0 \\ 0 & 0 & K_z \end{bmatrix}, \quad (25)$$

where $P = P(x, y, z)$ is the scalar pressure field, ∇P is the pressure gradient, μ is fluid viscosity, and $\mathbf{K}$ is the permeability tensor. Combining equations (24) and (25) gives

$$K_x \frac{\partial^2 P}{\partial x^2} + K_y \frac{\partial^2 P}{\partial y^2} + K_z \frac{\partial^2 P}{\partial z^2} = 0. \quad (26)$$

By using the transformed coordinates $x' = \frac{x}{\sqrt{K_x}}, y' = \frac{y}{\sqrt{K_y}}, z' = \frac{z}{\sqrt{K_z}}$, equation (26) becomes the following classical Laplace's equation

$$\nabla^2 P = \frac{\partial^2 P}{\partial x'^2} + \frac{\partial^2 P}{\partial y'^2} + \frac{\partial^2 P}{\partial z'^2} = 0, \quad (27)$$

where ∇^2 is the Laplacian operator. For a point well in an unbounded homogeneous domain, the transformed problem is spherically symmetric. Define the anisotropic radial coordinate

$$r^2 = x'^2 + y'^2 + z'^2 = \frac{x^2}{K_x} + \frac{y^2}{K_y} + \frac{z^2}{K_z}. \quad (28)$$

By using Cartesian to spherical coordinate transformation $x' = r \sin \psi \cos \theta$, $y' = r \sin \psi \sin \theta$, $z' = r \cos \psi$, and the pressure dependence only on r , the Laplace equation (27) in spherical coordinates can be rewritten as

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dP}{dr} \right) = 0,$$

which gives

$$P(r) = \frac{C_1}{r} + C_2, \quad (29)$$

where C_1 and C_2 are the integration constants. For $r \rightarrow \infty$, $P(r) \rightarrow P_{\text{ref}}$ (reference reservoir pressure), hence $C_2 = P_{\text{ref}}$ and

$$P(r) = P_{\text{ref}} + \frac{C_1}{r}. \quad (30)$$

The constant C_1 is obtained from the prescribed well rate Q , defined as in following the total Darcy flux through a closed surface S surrounding the well:

$$Q = \iint_S \mathbf{u} \cdot \mathbf{n} dS, \quad (31)$$

where $\mathbf{n}$ denotes outward unit vector in normal direction, and $\mathbf{u} \cdot \mathbf{n}$ is the part of Darcy velocity that actually crossing the surface. In an isotropic medium, a constant- r surface is spherical and $Q = 4\pi r^2 u_r$ where u_r is the radial Darcy velocity. However, in the anisotropic physical coordinates equation (28) defines an ellipsoid, so the general surface-flux form in Equation (31) is used.

Let $a = \sqrt{K_x}$, $b = \sqrt{K_y}$, and $c = \sqrt{K_z}$. The constant- r ellipsoid is parameterized by

$$x = ar \sin \psi \cos \theta, \quad y = br \sin \psi \sin \theta, \quad z = cr \cos \psi, \quad (32)$$

with outward vector surface element

$$\mathbf{n} dS = r^2 \begin{bmatrix} bc \sin^2 \psi \cos \theta \\ ac \sin^2 \psi \sin \theta \\ ab \sin \psi \cos \psi \end{bmatrix} d\psi d\theta. \quad (33)$$

From Equation (30),

$$\frac{\partial P}{\partial x} = -\frac{C_1 x}{K_x r^3}, \quad \frac{\partial P}{\partial y} = -\frac{C_1 y}{K_y r^3}, \quad \frac{\partial P}{\partial z} = -\frac{C_1 z}{K_z r^3}. \quad (34)$$

Using Darcy's law,

$$u_x = -\frac{K_x}{\mu} \frac{\partial P}{\partial x}, \quad u_y = -\frac{K_y}{\mu} \frac{\partial P}{\partial y}, \quad u_z = -\frac{K_z}{\mu} \frac{\partial P}{\partial z}, \quad (35)$$

By using equations (32) and (34) in (35), we obtain

$$\mathbf{u} = \frac{C_1}{\mu r^2} (a \sin \psi \cos \theta, b \sin \psi \sin \theta, c \cos \psi). \quad (36)$$

Substitution of equations (33) and (36) into Equation (31) gives

$$Q = \frac{C_1 abc}{\mu} \int_0^{2\pi} d\theta \int_0^\pi \sin \psi d\psi = \frac{4\pi C_1 abc}{\mu},$$

and hence

$$C_1 = \frac{Q\mu}{4\pi \sqrt{K_x K_y K_z}}. \quad (37)$$

The point-well pressure field equation (30) is therefore

$$P(r) = P_{\text{ref}} + \frac{Q\mu}{4\pi r \sqrt{K_x K_y K_z}}. \quad (38)$$

For a well located at (x_w, y_w, z_w) , define

$$R^2 = \frac{(x - x_w)^2}{K_x} + \frac{(y - y_w)^2}{K_y} + \frac{(z - z_w)^2}{K_z}.$$

Applying Darcy's law to Equation (38) gives

$$\mathbf{u} = \frac{Q}{4\pi\sqrt{K_x K_y K_z}} \frac{(x - x_w, y - y_w, z - z_w)}{R^3}. \quad (39)$$

For multiple wells, the linearity of the pressure equation allows superposition. Defining

$$R_i^2 = \frac{(x - x_{wi})^2}{K_x} + \frac{(y - y_{wi})^2}{K_y} + \frac{(z - z_{wi})^2}{K_z},$$

the total Darcy velocity is

$$\mathbf{u}(x, y, z) = \frac{1}{4\pi\sqrt{K_x K_y K_z}} \sum_{i=1}^n Q_i \frac{(x - x_{wi}, y - y_{wi}, z - z_{wi})}{R_i^3}, \quad (40)$$

or equivalently,

$$u_x = \frac{1}{4\pi\sqrt{K_x K_y K_z}} \sum_{i=1}^n \frac{Q_i(x - x_{wi})}{R_i^3}, \quad u_y = \frac{1}{4\pi\sqrt{K_x K_y K_z}} \sum_{i=1}^n \frac{Q_i(y - y_{wi})}{R_i^3},$$

$$\text{and } u_z = \frac{1}{4\pi\sqrt{K_x K_y K_z}} \sum_{i=1}^n \frac{Q_i(z - z_{wi})}{R_i^3}.$$

Here, n is the number of wells, Q_i is the volumetric rate of well i , and (x_{wi}, y_{wi}, z_{wi}) is its location. Positive Q_i denotes an injector and negative Q_i a producer.

B Derivation of baseline resistance

Equation (6) gives the initial resistance over the available traced portion of streamline i as

$$\hat{R}_i(0) = \sum_b r_{i,b}(0), \quad r_{i,b}(0) = \frac{l_{i,b}}{K_{\text{eff},i,b} A_{i,b} \lambda_T(S_{w,\text{init}})}, \quad (41)$$

where $S_{w,\text{init}}$ is the initial water saturation and the remaining notation is defined in Subsection 2.5. For injector j , each launched streamline is assigned the same initial reference flow q_0 . If $N_{c,j}$ streamlines are captured, the total reference flow represented by the captured streamlines set C_j is

$$Q_{\text{cap},j}^0 = N_{c,j} q_0. \quad (42)$$

The captured streamlines form parallel hydraulic paths between the injector and producer. Their equivalent initial resistance and corresponding reference pressure drop are

$$R_{\text{eq},\text{cap},j}^0 = \left[\sum_{i \in C_j} \frac{1}{\hat{R}_i(0)} \right]^{-1}, \quad \Delta p_{0,j} = Q_{\text{cap},j}^0 R_{\text{eq},\text{cap},j}^0. \quad (43)$$

Using Equation (42),

$$\Delta p_{0,j} = N_{c,j} q_0 \left[\sum_{i \in C_j} \frac{1}{\hat{R}_i(0)} \right]^{-1}. \quad (44)$$

Then a common baseline resistance is defined in following expression so that a streamline carrying the reference flow q_0 experiences the same reference pressure drop:

$$R_{\text{base}} = \frac{\Delta p_{0,j}}{q_0} = \frac{N_{c,j}}{\sum_{i \in C_j} \frac{1}{\hat{R}_i(0)}}. \quad (45)$$

Thus, R_{base} is the harmonic mean of the initial traced resistances of the captured streamlines. This common baseline is assigned to all launched streamlines from injector j before saturation-induced resistance changes are applied.

C Physical interpretation of the near-well skin factor

The skin term in equation (16) can be interpreted by considering an altered near-well region extending from the wellbore radius r_w to an outer radius r_s . Let its permeability be K_s , while the undisturbed reservoir between r_s and the streamline cutoff radius r_c has permeability K_h . Assuming near-well mobility λ_{nw} , radial Darcy flow gives

$$d(\Delta p) = \frac{q}{2\pi h \lambda_{\text{nw}}} \frac{dr}{K(r)r}. \quad (46)$$

Hence, the pressure loss from r_w to r_c is

$$\Delta p_{\text{nw}} = \frac{q}{2\pi h \lambda_{\text{nw}}} \left[\int_{r_w}^{r_s} \frac{dr}{K_s r} + \int_{r_s}^{r_c} \frac{dr}{K_h r} \right], \quad (47)$$

which gives

$$\Delta p_{\text{nw}} = \frac{q}{2\pi K_h h \lambda_{\text{nw}}} \left[\frac{K_h}{K_s} \ln \left(\frac{r_s}{r_w} \right) + \ln \left(\frac{r_c}{r_s} \right) \right]. \quad (48)$$

Using $\ln(r_c/r_w) = \ln(r_s/r_w) + \ln(r_c/r_s)$, this becomes

$$\Delta p_{\text{nw}} = \frac{q}{2\pi K_h h \lambda_{\text{nw}}} \left[\ln \left(\frac{r_c}{r_w} \right) + \left(\frac{K_h}{K_s} - 1 \right) \ln \left(\frac{r_s}{r_w} \right) \right]. \quad (49)$$

The dimensionless skin factor is therefore

$$S = \left(\frac{K_h}{K_s} - 1 \right) \ln \left(\frac{r_s}{r_w} \right), \quad \Delta p_{\text{nw}} = \frac{q}{2\pi K_h h \lambda_{\text{nw}}} \left[\ln \left(\frac{r_c}{r_w} \right) + S \right]. \quad (50)$$

Thus, $K_s < K_h$ gives $S > 0$, $K_s = K_h$ gives $S = 0$, and $K_s > K_h$ gives $S < 0$. In the present streamline model, K_s and r_s are not represented explicitly; instead, S is used as an effective near-well correction parameter in equation (16).